Software Reliability Growth Modeling Involving Burr Type XII distribution and Fault Removal Efficiency

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Abstract

In software reliability analysis various authors have used Burr type XII distribution to model the failure pattern of the system due to its wide variety of flexible shapes. In particular cases it can be reduced to Exponential, Normal, Weibull, Log-logistic, Gamma distributions etc. In proposed paper software reliability growth model has been developed incorporating fault removal efficiency (FRE) and Burr type XII based testing effort function. FRE represents fraction of detected faults which are removed completely. Parameters of model are predicted by LSE whereas MSE is used to perform comparison analysis. Results validate better fitting of data set.

Keywords: Burr type XII TEF, FRE, NHPP, SRGM.

1. INTRODUCTION

Reliability of software system is measured in terms of probability that software performs desired operations without failure for specific time duration known as testing time. Reliability depends on number of faults detected and corrected during testing time phase. Various mathematical models have been developed to formulate expression for mean number of faults $m(t)$ detected in time interval $(0, t)$. Such models are known as Software Reliability Growth Model’s (SRGM). Software Reliability is the conditional probability that no faults detected in time interval $(t, t + \Delta t)$ and formulated as

$$R(t) = P \left( \frac{x}{t} \right) = \exp( -\left[ m(t + \Delta t) - m(t) \right] )$$

Broadly there are two types of SRGM, Markovian model and Fault counting model. Markovian process is used to represents faults in Markovian model. In Fault counting model failure distribution is described by Homogenous Poisson Process (HPP), Non-Homogenous Poisson Process (NHPP) etc.

Test Effort Function (TEF) is one of the various factors which influence reliability of software. It measures expenditure incurred on man power used, CPU working hour, number of test cases used, etc. Several SRGM’s have been formulated involving different types of TEF [1-3], [5], [7-14]. Aggarwal et al.[12] incorporated various TEF into the model developed using modular approach. They classified faults as simple, hard and complex. An optimization problem has been formulated of maximizing faults removed subject to budgetary and reliability constrains. Genetic algorithm has been used to solve the problem. Many authors have used Burr type distributions to measure testing efforts. Ahmad et al.[15] proposed model involving Burr Type X TEF and determined optimum release time. Bokhari and Ahmad[16] used Burr Type XII TEF into SRGM and analyzed real data collected by Rome Air Development Centre. They also proposed optimum release policy of software system depending on reliability and cost criterion.

Fault removal efficiency (FRE) is another factor which affects the reliability of software. Fault removal process is very complex and time-consuming process. It involves detection and correction of faults. Many SRGM’s assumes faults detected are removed completely during debugging process. It is possible some faults are left in software system. Fault removal efficiency is the fraction of removed faults in first repair attempt. Various authors [17-18] have developed SRGM involving FRE.

In this paper SRGM has been developed involving Burr type XII TEF and FRE. After introduction there are four sections. In Section 2 Burr type XII distribution and its properties are discussed. Section 3 presents development of model and solution. Section 4 provides estimation of the parameter and comparison using statistical tools. Finally, conclusion has been highlighted in section 5.

2. BURR TYPE XII DISTRIBUTION

I. W. Burr [20] developed twelve probability distributions collectively known as Burr system. Burr type XII is the most important distribution which has two positive shape parameters. It is widely applied for modeling in reliability analysis, survival analysis, actuaries, economics, forestry, hydrology and meteorology due to its probability density function which
exhibits either decreasing or unimodal nature. Let $T$ be a random variable which denotes the failure time of an item and $F(t)$ is the probability that item fails before or at time $t$. Burr type XII distribution of $F(t)$ is defined as
\[ F(t) = \text{Probability}(T \leq t) = 1 - (1 + t^\alpha)^{-\beta} , \quad t > 0 \] (1)

Where $\alpha, \beta$ are shape parameters.

If $f(t)$ represents density function then
\[ f(t) = \frac{dF(t)}{dt} = \alpha \beta t^{\alpha-1}(1 + t^\alpha)^{-\beta-1} , \quad t > 0 \] (2)

Under fixed $\beta$, $f(t)$ is decreasing for $0 < \alpha \leq 1$ and unimodal for $\alpha > 1$.

Hazard rate function $h(t)$ is given by
\[ h(t) = \alpha \beta t^{\alpha-1}(1 + t^\alpha)^{-\beta} , \quad t > 0 \] (3)

Mean and variance are given by
\[ \mu = E(T) = \frac{\Gamma\left(\beta - \frac{1}{\alpha}\right) \Gamma\left(\frac{1}{\alpha}\right)}{\Gamma(\beta)} , \quad \alpha \beta > 1 \] (4)
\[ \sigma^2 = \frac{\Gamma\left(\beta - \frac{2}{\alpha}\right) \Gamma\left(\frac{2}{\alpha}\right)}{\Gamma(\beta)} - \mu^2 , \quad \alpha \beta > 2 \] (5)

3. MODEL DEVELOPMENT

Let $NF(t)$ be the number of faults detected by the time $t$. Using Poisson distribution probability of detecting $r$ faults by the time $t = t_1$ is given by
\[ P(NF(t = t_1) = r) = \frac{m(t_1)^r \exp(-m(t_1))}{r!} \]

$m(t)$ is given by differential equation
\[ \frac{dm(t)}{dt} = b(a - m(t)) , \quad 0 < b < 1, \quad a, t > 0 \] (6)

Where $b$ is fault detection rate and $a$ is total number of faults.

Introducing fault removal factor $p$ in equation (6), we get
\[ \frac{dm(t)}{dt} = b(a - pm(t)) \] (7)

Term $pm(t)$ is the fraction of removed faults at time $t$. $p$ lies between 0 and 1, if it is 1 then complete removal of detected faults.

Let $T_F(t)$ be the Burr type XII TEF and it has been modified in software reliability analysis as
\[ T_F(t) = E(1 - (1 + (st)^\alpha)^{-\beta}) , \quad E,s,t,\alpha,\beta > 0 \] (8)

Where $E$ is the total amount of test effort expenditure and $s$ is the scale parameter. Density function $t_f(t)$ is given by
\[ t_f(t) = E\alpha s E(1 + (st)^\alpha)^{-\beta-1} \] (9)

Incorporating $tf(t)$ in equation (7), we get
\[ \frac{dm(t)}{dt} = b(a - pm(t))(t_f(t)) \] (10)

Using condition $m(0) = 0$ and $T_F(0)=0$, solution of equation (10) is
\[ m(t) = \frac{\alpha}{p}(1 - \exp(-pE\alpha s t)) \] (11)

4. PARAMETER ESTIMATION AND COMPARISON

Least Squares Estimation method is used to predict the values of parameters using CSR3 data set compiled by M.R. Lyu [21]. Proposed model is compared with Bokhari and Ahmad [16] model using statistical tool mean square error (MSE).

\[ MSE = \sum_{i=1}^{n}(\hat{m}(t) - \hat{m}(t))^2 / n \]

Where $\hat{m}(t)$ and $\hat{m}(t)$ are predicted and actual cumulative faults at time $t$ respectively. $n$ is number of data points. Lower the value of MSE better is the fitting.

<table>
<thead>
<tr>
<th>Model</th>
<th>MSE</th>
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<tbody>
<tr>
<td>Proposed Model</td>
<td>42.17</td>
</tr>
<tr>
<td>Bokhari and Ahmad [16] model</td>
<td>53.12</td>
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5. CONCLUSION
In proposed model mean number of faults are forecasted in which test efforts expenditure represented by Burr type XII probability distribution. While developing model it is also assumed that only fraction of faults detected are removed completely. On comparing with Bokhari and Ahmad [16] model, it has been found that the proposed model fits data set better.

REFERENCES