A Systematic and Extensive Determination of Far Field Time Harmonic Electromagnetic Field of a Radiating Element

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Abstract

Maxwell's equations have been used in time domain analysis, and it has been successful in a wide range of applications. Writing computer code to solve a straightforward scattering issue requires minimal effort than conventional frequency domain methods. However, issues arise when the time domain approach is employed to resolve arbitrarily complex problems. The simple time domain algorithm is not adequate for many real-world problems. For appropriate excitation in the time domain, for example, a waveguide transition analysis should have information on the incoming and outgoing mode patterns. If there is a time-varying current, then there is a possibility that it may produce radiation. In cases like this, frequency and time domain analysis is necessary, and their combination will produce an effective outcome. This paper provides a systematic and extensive determination of far-field time harmonic electromagnetic field, power flow, and radiation pattern in a desired direction of a radiating element.

Introduction

Time harmonic fields are those electro-dynamic fields that vary periodically and sinusoidally with time [1]. The need for effective numerical solutions for the Maxwell equations in the frequency domain is central to many theoretical and practical issues in electromagnetic. These equations have a vector structure, so the representations frequently require complex expressions, like series over the vector spherical harmonics. This causes an excessive amount of unknowns in function representations as well as complex formulas, both of which can be sources of error. However, researchers have created a theory for the translation of such series with vector spherical harmonics and techniques to calculate the translation coefficients despite these challenges [2-5]. Every solution of the free-space Maxwell equations can be represented by two scalar potentials, which are answers to the scalar Helmholtz equation [6,7], which are also called the Debye potentials [8]. Both function representations using multipole-type series and the far field signature function have reasonably well-developed translation theories for the Helmholtz equation [9-11]. However, several problems need to be resolved to apply this translation theory to the scalar potential representation of Maxwell equation solutions. The objective of this paper is to present such a theory.

The method that we present in this paper is similar to the translation of biharmonic equation solutions that was previously developed [12], where it was demonstrated that any biharmonic equation solution can be expressed as a combination of two solutions of the Laplace equations. In staying with that paper's methodology, we present a potential representation, the idea of "conversion" operators, and demonstrate how, given a multipole routine for the scalar Helmholtz equation, a routine for the vector Maxwell equations can be created by employing the conversion operators. There is no doubt that these conversion operators vary from those used for the biharmonic equation. With vector representations, keeping the solution's absence of divergence is a significant challenge that must be overcome.

1. Mathematical Models:

The electromagnetic phenomenon is analyzed under macroscopic view where, linear dimension is larger than the atomic dimension and charge magnitude is greater than the atomic charges.

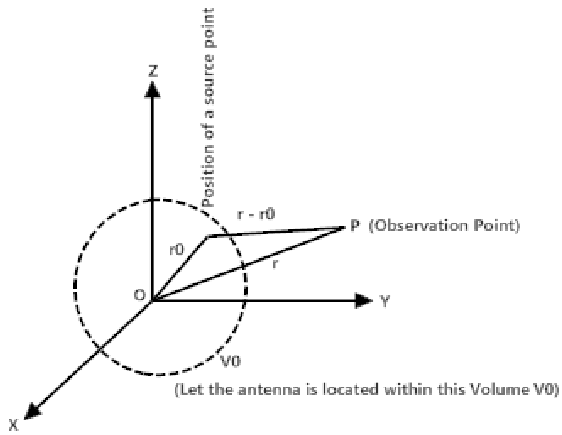

Fig.1: The co-ordinate of the observation point

Calculation of radiated fields from an antenna:

Let's assume that $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$

$$\nabla X\, \vec{E} = \frac{-\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{1}$$

$$\nabla X\, \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \tag{2}$$

$$\nabla . \vec{B} = 0 \Rightarrow \nabla . \vec{H} = 0 \tag{3}$$

$$\nabla . \vec{D} = \rho_e \Rightarrow \nabla . \vec{E} = \frac{\rho_e}{\varepsilon} \tag{4}$$

Maxwell's equation in frequency domain will be:

$$\nabla X\, \vec{E}\, (\vec{r}) = -j\omega\mu_0 \vec{H}(\vec{r}) \tag{5}$$

$$\nabla X\, \vec{H}\, (\vec{r}) = \sigma \vec{E}(\vec{r}) + j\omega\varepsilon_0 E(\vec{r}) = j\omega\varepsilon_0 E(\vec{r}) + \overrightarrow{Je}(\vec{r}) \tag{6}$$

$$\nabla . \vec{H}(\vec{r}) = 0 \tag{7}$$

$$\nabla . \vec{E}(\vec{r}) = \frac{\rho e(\vec{r})}{\varepsilon_0} \tag{8}$$

Where, $\vec{E}$ is the electric intensity in volts per meter, $\vec{H}$ is the magnetic intensity in amperes per meter, $\vec{D}$ is the electric flux density in coulombs per square meter, $\vec{B}$ is the magnetic flux density in webers per square meter, ρ_e is the electric charge density in coulombs per cubic meter, and $\overrightarrow{Je}$ electric current density amperes per square meter.

2. Procedure to solve the Maxwell's equation:

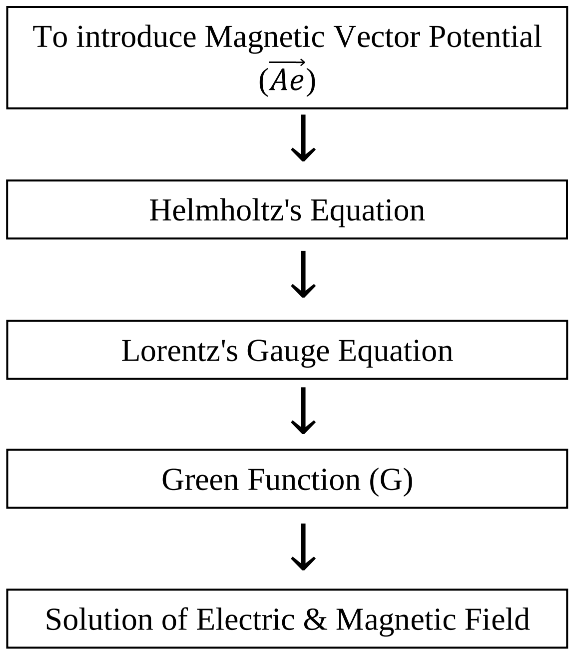

Fig.2: Steps to solve the Maxwell's equation

Introduction of Magnetic Vector Potential:

From the Maxwell's Equation

$$\nabla . \vec{B} = 0 \tag{9}$$

we know divergence of curl is zero

$$\nabla . (\nabla X \overrightarrow{A_e}) = 0 \tag{10}$$

From equation (9) and (10)

$$\vec{H} = \frac{1}{\mu_0}(\nabla X \overrightarrow{A_e}) \{\overrightarrow{A_e} = \text{Magnetic Vector Potential}\} \tag{11}$$

$$\Rightarrow \nabla X \overrightarrow{A_e} = \mu_0 \vec{H}$$

$$\Rightarrow \nabla X \nabla X \overrightarrow{A_e} = \mu_0 (\nabla X \vec{H}) = \mu_0 [j\omega\varepsilon_0 \vec{E} + \overrightarrow{Je}] \tag{12}$$

$$\begin{cases}\nabla X \vec{E} = -j\omega\mu_0\vec{H} \\ \vec{H} = \frac{1}{\mu_0}(\nabla X \overrightarrow{A_e}) \end{cases}\} \vec{E} = -j\omega A_e - \nabla\emptyset_e \quad (13)$$

where $\nabla\emptyset_e$ is the scalar potential function and is unknown.

Putting Equation (13) in (12) we get :

$$\nabla X \ \nabla X \overrightarrow{A_e} = \mu_0\varepsilon_0 j\omega[-j\omega\overrightarrow{A_e} - \nabla\emptyset_e] + \mu_0\overrightarrow{Je} \quad (14)$$

$$\Rightarrow \nabla X \ \nabla X \ \overrightarrow{A_e} = \omega^2\mu_0\varepsilon_0\overrightarrow{A_e} - j\omega\mu_0\varepsilon_0 (\nabla\emptyset_e) + \mu_0\overrightarrow{Je} = K_0^2\overrightarrow{A_e} - j\omega\mu_0\varepsilon_0 \nabla\emptyset_e + \mu_0\overrightarrow{Je} \quad (15)$$

From the vector identities, we know $\nabla X \ \nabla X \overrightarrow{A_e} = \nabla(\nabla . \overrightarrow{A_e}) - \nabla^2 \overrightarrow{A_e}$ (16)

$$\Rightarrow \nabla(\nabla . \overrightarrow{A_e}) - \nabla^2 \overrightarrow{A_e} = K_0^2\overrightarrow{A_e} - j\omega\mu_0\varepsilon_0 \nabla\emptyset_e + \mu_0\overrightarrow{Je}$$

$$\Rightarrow K_0^2\overrightarrow{A_e} + \nabla^2 \overrightarrow{A_e} = \nabla(\nabla . \overrightarrow{A_e}) + j\omega\mu_0\varepsilon_0 \nabla\emptyset_e - \mu_0\overrightarrow{Je} \quad (18)$$

According to Lorentz's Gauge Equation: $j\omega \mu_0\varepsilon_0 \nabla\emptyset_e = -\nabla\overrightarrow{A_e}$ (19)

$$K_0^2\overrightarrow{A_e} + \nabla^2 \overrightarrow{A_e} = \nabla(\nabla . \overrightarrow{A_e}) + j\omega\mu_0\varepsilon_0 \nabla\emptyset_e - \mu_0\overrightarrow{Je} \quad (20)$$

Applying Lorentz's Gauge Equation:

$$\nabla(\nabla . \overrightarrow{A_e}) + j\omega\mu_0\varepsilon_0 \nabla\emptyset_e \approx 0 \quad (21)$$

Therefore, $K_0^2\overrightarrow{A_e} + \nabla^2 \overrightarrow{A_e} = -\mu_0\overrightarrow{Je}$ (22)

The obtained equation is Helmholtz's equation or wave equation. If electromagnetic theory is a solar system , then Helmholtz's equation is the sun. It controls almost everything. Now we need to consider response to a point source and that brings us to Green's Function.

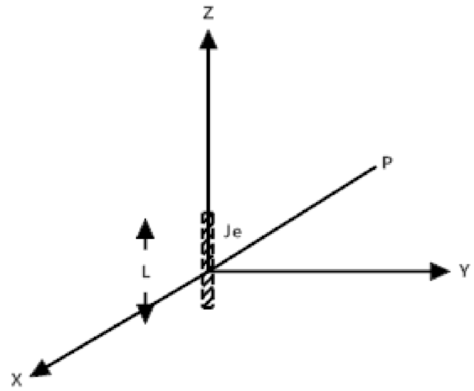

Fig. 3: The current distribution in a dipole

Let that point source is located at the origin of the co-ordinate system. The current distribution along the electric dipole will be :

$$\overrightarrow{Je} = I0\rho\delta(x)\delta(y)\delta(z)\overrightarrow{u_z} \qquad (23)$$

As current is along the $\vec{Z}$ axis the magnetic vector potential will be along the "z" axis.$[\overrightarrow{A_e} = A_{ez}\overrightarrow{u_z}]$

$$\Rightarrow \overrightarrow{Je} = I_0\rho\delta(\vec{r})\overrightarrow{u_z} \qquad (24)$$

For low frequency operation, length of the antenna is much more lesser than the wavelength(λ).

Here, I0ρ is called the Dipole Moment.

Customizing Helmholtz's equation :

$$\nabla^2\overrightarrow{A_e} + \overrightarrow{K_0 A_e} = -\mu_0\overrightarrow{Je} \qquad (25)$$

$$\nabla^2\overrightarrow{A_{ez}} + K_0^2\overrightarrow{A_{ez}} = -\mu_0 I_0\rho\delta(\vec{r}) \qquad (26)$$

where $\nabla^2\overrightarrow{A_{ez}}$ representing in terms of spherical co-ordinate system.

$$\frac{1}{r^2}\frac{d}{dr}[r^2\frac{dA_{ez}}{dr}] + K_0^2 A_{ez} = \mu_0 I_0\rho\delta(\vec{r}) \qquad (27)$$

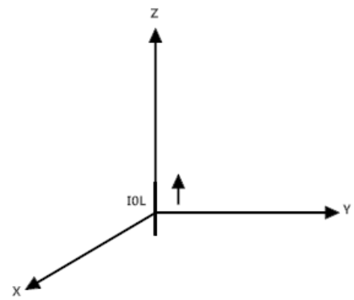

Fig.4: The dipole moment

There are two solutions for the differential equation :

$$A_{ez} = \begin{cases} \frac{c}{r} e^{jk_0 r} \\ \frac{c}{r} e^{-jk_0 r} \end{cases} \quad (28)$$

The upper part of equation (28) implies that the wave is moving towards the source. The bottom part of the equation implies that the wave is moving towards the load and away from the source. This first solution does not exist in a real physical world. Therefore the second solution is the real solution.

$$A_{ez} = \frac{c}{r} e^{-jk_0 r} \quad (29)$$

where, c = constant → It is required to find the value of "c". To find "c", let's assume w = 0 and k = 0. [as w = 0 and k = 0, it is a static condition. Therefore, at w= 0 and k = 0, electromagnetic equation becomes Poisson's equation. The vector potential

$A_{ez} = \frac{\mu_0 I_0 \rho}{4\pi r} \Rightarrow$ Therefore $c = \frac{\mu_0 I_0 \rho}{4\pi r}$. The only component exist is in z direction.

So,

$$\overrightarrow{A_e} = A_{ez}\overrightarrow{uz} = \frac{\mu_0 I_0 \rho}{4\pi r} e^{-jk_0 r} \overrightarrow{u_z} \quad (30)$$

$$\overrightarrow{A_e} = \frac{\mu_0 I_0 \rho}{4\pi} \frac{e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}}{|\vec{r}-\vec{r}0|} \quad (31)$$

$$\overrightarrow{A_e} = \frac{\mu_0}{4\pi} \iiint \frac{\overrightarrow{\vec{J}e(\overrightarrow{r_0})} e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}}{|\vec{r}-\vec{r}0|} \quad (32)$$

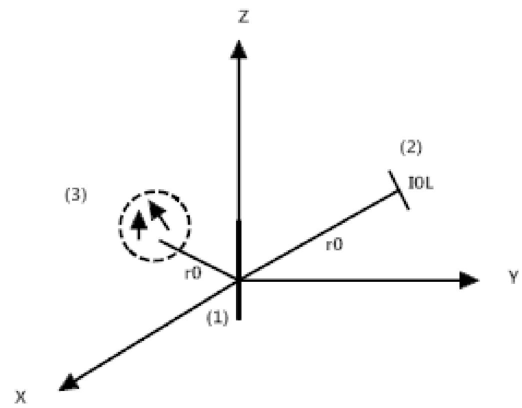

Fig.4: The magnetic vector potential of an arbitrarily distributed current

The solution of magnetic vector potential of an arbitrarily distributed current in an antenna.

$$\overrightarrow{A_e} = \iiint \overrightarrow{Je}(\overrightarrow{r_0})\left[\frac{\mu_0 e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}}{4\pi|\vec{r}-\overrightarrow{r_0}|}\right]dv_0 \quad (33)$$

$$\vec{H} = \frac{1}{\mu_0}(\nabla X \overrightarrow{Ae}) \quad (34)$$

Calculation of electric and magnetic field in terms of magnetic vector potential :

From the general solution of field , we know :

$$\vec{H} = \frac{1}{\mu_0}(\nabla X \overrightarrow{Ae})$$

$$\vec{E} = -j\omega\overrightarrow{Ae} - \nabla\emptyset_e$$

where $\nabla\emptyset_e$ is the scalar potential. The scalar potential can be written in terms of vector potential by applying Lorentz's Gauge equation.

$$\nabla X \overrightarrow{A_e} = -j\omega\varepsilon_0\mu_0\emptyset_e \quad (35)$$

$$\Rightarrow \nabla(\nabla.\overrightarrow{A_e}) = -j\omega\varepsilon_0\mu_0(\nabla\emptyset_e) \quad (36)$$

$$\Rightarrow \nabla\emptyset_e = \frac{-\nabla(\nabla.\overrightarrow{Ae})}{j\omega\varepsilon_0\mu_0} \quad (37)$$

Again applying vector identity:

$$\nabla^2\overrightarrow{A_e} = \nabla(\nabla.\overrightarrow{A_e}) - \nabla X\ \nabla X\overrightarrow{A_e} \Rightarrow \nabla(\nabla.\overrightarrow{A_e}) = \nabla^2\overrightarrow{A_e} + \nabla X\ \nabla X\overrightarrow{A_e}$$

As

$$\vec{E} = -j\omega\overrightarrow{A_e} - \nabla\emptyset_e = -j\omega\overrightarrow{A_e} + \frac{\nabla(\nabla.\overrightarrow{A_e})}{j\omega\varepsilon_0\mu_0} = \frac{-j^2\omega^2\mu_0\varepsilon_0\overrightarrow{A_e} + \nabla(\nabla.\overrightarrow{A_e})}{j\omega\varepsilon_0\mu_0} = \frac{\omega^2\mu_0\varepsilon_0\overrightarrow{A_e} + \nabla(\nabla.\overrightarrow{A_e})}{j\omega\varepsilon_0\mu_0}$$

$$= \frac{K_0^2\overrightarrow{A_e} + \nabla(\nabla.\overrightarrow{A_e})}{j\omega\varepsilon_0\mu_0} = \frac{K_0^2\overrightarrow{A_e} + \nabla^2\overrightarrow{A_e} + \nabla X \nabla X \overrightarrow{A_e}}{j\omega\varepsilon_0\mu_0}$$

Thus,

$$\vec{E} = \frac{1}{j\omega\varepsilon_0\mu_0}[\nabla X \ \nabla X \overrightarrow{A_e} + \nabla 2\overrightarrow{A_e} + K_0^2\overrightarrow{A_e}] \qquad (38)$$

$$\Rightarrow \vec{E} = \frac{1}{j\omega\varepsilon_0\mu_0}[\nabla X \ \nabla X \overrightarrow{A_e} - \mu_0\overrightarrow{Je}] \qquad (39)$$

Beyond the source zone or at far away from the source $\overrightarrow{Je} = 0$.

Therefore, $\vec{E} = \frac{1}{j\omega\varepsilon_0\mu_0}[\nabla X \ \nabla X \overrightarrow{A_e}]$

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\varepsilon_0\mu_0}[\nabla X \ \nabla X \overrightarrow{A_e}] \qquad (40)$$

Putting the value of $\overrightarrow{A_e}$ in (40)

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\varepsilon_0\mu_0}[\nabla X \ \nabla X \iiint \overrightarrow{Je}(\overrightarrow{r_0}) \frac{\{\mu_0 e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}}{4\pi|\vec{r}-\overrightarrow{r_0}|}\} dv_0 \qquad (41)$$

Calculation of Radiated Field

At the Initial stage we have magnetic vector potential.

$$A_e(\vec{r}) = \iiint \overrightarrow{Je}(\overrightarrow{r_0}) \frac{e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}}{4\pi|\vec{r}-\overrightarrow{r_0}|} dv_0 \qquad (42)$$

$$\vec{H}(\vec{r}) = \frac{1}{\mu_0}\nabla X \overrightarrow{A_e}(\vec{r}) \qquad (43)$$

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\varepsilon_0\mu_0}[\nabla X \ \nabla X \overrightarrow{A_e}(\vec{r})] \qquad (44)$$

Considering Equation (42) and (43) we got :

$$\vec{H}(\vec{r}) = \frac{1}{\mu_0}\nabla X \iiint \overrightarrow{Je}(\overrightarrow{r_0}) \frac{e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}}{4\pi|\vec{r}-\overrightarrow{r_0}|} dv_0$$

$$\vec{H}(\vec{r}) = \frac{1}{\mu_0}\iiint \overrightarrow{Je}(\overrightarrow{r_0})\varphi(\vec{r}, \overrightarrow{r_0}) dv_0 \qquad (45)$$

Where, $\varphi(\vec{r}, \overrightarrow{r_0}) = \frac{e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}}{4\pi|\vec{r}-\overrightarrow{r_0}|}$

Step-1

$$\nabla_r X \overrightarrow{Je}(\overrightarrow{r_0})\varphi(\vec{r}, \overrightarrow{r_0}) = \mu(\vec{r}, \overrightarrow{r_0}) \nabla_r X \overrightarrow{Je}(\overrightarrow{r_0}) + \nabla_r\varphi(\vec{r}, \overrightarrow{r_0}) X \overrightarrow{Je}(\overrightarrow{r_0})$$

$$Thus\ ,\ \nabla_r\ X\ \overrightarrow{Je}(\overrightarrow{r_0})\ \varphi(\vec{r}\ ,\overrightarrow{r_0}) = \nabla_r \varphi\overrightarrow{(r}\ ,\overrightarrow{r_0})\ X\ \overrightarrow{Je}(\overrightarrow{r_0}) \qquad (46)$$

$$\nabla_r \varphi\overrightarrow{(r}\ ,\overrightarrow{r_0}) = \frac{1}{4\pi}\nabla_r\{\frac{1}{|\vec{r}-\overrightarrow{r_0}|}\}\ e^{-jk_0|\vec{r}-\overrightarrow{r_0}|} + \frac{1}{4\pi}\frac{1}{|F-F_0|}\nabla_r\{\ e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}\}$$
(47)

$$\nabla_r\{\frac{1}{|\vec{r}-\overrightarrow{r_0}|}\} = \frac{(\vec{r}-\overrightarrow{r_0})}{|\vec{r}-\overrightarrow{r_0}|}$$

$$\nabla_r\{\ e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}\} = -jk_0\ e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}\ \frac{(\vec{r}-\overrightarrow{r_0})}{|\vec{r}-\overrightarrow{r_0}|}$$

substituting in (47)

$$\nabla_r \varphi\overrightarrow{(r}\ ,\overrightarrow{r_0}) = -\frac{1}{4\pi}\frac{(\vec{r}-\overrightarrow{r_0})}{|\vec{r}-\overrightarrow{r_0}|}\ e^{-jk_0|\vec{r}-\overrightarrow{r_0}|}\{\frac{1}{|\vec{r}-\overrightarrow{r_0}|} + \frac{jk_0}{|\vec{r}-\overrightarrow{r_0}|}\} = -\frac{1}{4\pi}\frac{\overrightarrow{r_d}}{r_d}\ e^{-jk_0 r_d}\{\frac{1}{r_d^2} +\ j\ \frac{k0}{r_d}\}$$
(48)

Step-2 : We know

$$\vec{H}\ (\vec{r}) = \iiint \nabla_r\ X\ \overrightarrow{Je}(\overrightarrow{r_0})\varphi(\vec{r}\ ,\overrightarrow{r_0}\ )dv_0$$

Therefore, $\nabla_r\ X\ \overrightarrow{Je}(\overrightarrow{r_0})\ \varphi(\vec{r}\ ,\overrightarrow{r_0}) = -\ \frac{1}{4\pi}(jk_0 + \frac{1}{r_d})\frac{e^{jk_0k_d}}{r_d}\ \vec{u}_{r_d}\ X\ \overrightarrow{Je}(\overrightarrow{r_0})$ *(49)*

where , $r_d = |\vec{r} - \overrightarrow{r_0}|$

Combining these above equations :

$$\vec{H}\ (\vec{r}) = -\frac{1}{4\pi}\iiint (jk_0 + \frac{1}{r_d})\frac{e^{-jk_0r_d}}{r_d}\vec{u}_{r_d}\ X\ \overrightarrow{Je}(\overrightarrow{r_0})\ dv_0 \qquad (50)$$

Let's assume $|\vec{r}| >> |\overrightarrow{r_0}| = jk_0$

Therefore, $\vec{H}\ (\vec{r}) = -\ \frac{jk_0}{4\pi r}\vec{u}_r\ X \iiint \overrightarrow{Je}(\overrightarrow{r_0})e^{-jk_0r_d})\ dv_0$ *(51)*

$$r_d = \sqrt{(x-\ x_0)^2 + (y-\ y_0)^2 + (z-\ z_0)^2}$$

$$= \sqrt{x^2 + y^2 + z^2 - 2(xx_0 + yy_0 + zz_0) +\ x_0^2 + y_0^2 + z_0^2}$$

$$= {}^r\sqrt{1 - 2(\frac{xx_0+yy_0+zz_0}{r^2}) + \frac{r_0^2}{r^2}}$$

$$= r\ [1 + \frac{1}{2}(\frac{-2}{r^2}(xx_0 + yy_0 + zz_0)) + \frac{r_0^2}{r^2} - \frac{1}{\delta}(.)\ \ldots\ldots]$$

Therefore,

$$r_d = r[1 - \frac{xx_0+yy_0+zz_0}{r^2} + \frac{r_0^2}{2r^2} - (.)] \approx r - \frac{\vec{r}}{r} . \overrightarrow{r_0} = r - \vec{u}_r . \overrightarrow{r_0} \quad (52)$$

Now

$$\vec{H}(\vec{r}) = - \frac{jk_0 e^{-jk_0 r}}{4\pi r} \vec{u}_r X \iiint \overrightarrow{Je}(\overrightarrow{r_0}) e^{-jk_0 r_0}) dv_0 \quad (53)$$

where , 4πr represents spherical expansion of the wave, while propagating in r direction and $\vec{H}(\vec{r})$ field has no radial component. The $\vec{H}$ field with no radial component will be :

$$\vec{H}(\vec{r}) = \frac{e-jk_0 r}{r}(\vec{\mu}_\theta H_\theta(\theta , \Phi) + \vec{\mu}_\Phi H_\Phi(\theta , \Phi) \quad (54)$$

Step 3

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\varepsilon_0\mu 0}[\nabla X \nabla X \overrightarrow{A_e}(\vec{r})] \quad (55)$$

For Field approximation: For Far Field r > $\frac{2}{\lambda 0}D^2$

$$\vec{E}(\vec{r}) = - \frac{k_0^2}{j\omega\varepsilon_0} \frac{e^{-jk_0 r}}{4\pi r} \vec{\mu}_r X \vec{\mu}_r X \iiint \overrightarrow{Je}(\overrightarrow{r_0}) e^{jk_0} \vec{\mu}_r \vec{r}_0 dv_0 \quad (56)$$

With no radial component

$$\vec{E}(\vec{r}) \frac{e^{-jk_0}}{r}(\vec{\mu}_\theta E_\theta (\theta , \Phi) + \vec{\mu}_\emptyset E_\emptyset (\theta , \Phi) \quad (57)$$

3. Generalized far field equation for the electric ($\overrightarrow{E}$) and magnetic ($\overrightarrow{H}$) field

The far field expression for the electric field will be:

$$\vec{E}(\vec{r}) = - \frac{k_0^2}{j\omega\varepsilon_0} (\frac{e^{-jk_0 r}}{4\pi r}) \vec{\mu}_r X \vec{\mu}_r X \iiint \overrightarrow{Je}(\overrightarrow{r_0}) e^{jk_0} \vec{\mu}_r \vec{r}_0 dv_0 \quad (58)$$

The far field expression for the magnetic field will be:

$$\vec{H}(\vec{r}) = - \frac{jk_0 e^{-jk_0 r}}{4\pi r} \vec{\mu}_r X \iiint \overrightarrow{Je}(\overrightarrow{r_0}) e^{jk_0} \vec{\mu}_r \vec{r}_0 dv_0 \quad (59)$$

The time average Poynting vector specifying power flow in a particular direction 'r' will be

$$S_p(\vec{r}) = \frac{1}{2} Re[\vec{E}(\vec{r}) \times \vec{H}(\vec{r})] \quad (60)$$

Putting the value of $\vec{E}(\vec{r})$ from equation (58) and $\vec{H}(\vec{r})$ from the equation (60) we can calculate the total power flow. Therefore radiation power per element of solid angle will be:

$$F(\theta, \emptyset) = \frac{|r^2 S_p \overrightarrow{(r)}|}{P_{max}} \quad (61)$$

Conclusion

In this paper, we discussed the method for solving time-harmonic Maxwell equations for the far electric and magnetic fields of an antenna. The final equation is customized according to the boundary condition, the presence of finite or infinite ground plane, and the dimension of the radiator to calculate the fundamental parameters like the radiation pattern, the total radiated power, directive gain, input impedance, radiation resistance, etc. The vector function operations are converted to scalar potential operations by modifying the translation theory.

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