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Some Curvature Properties on a Generalized Contact Metric Structure Manifold

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Abstract

In the present paper, We have defined generalized contact metric structure manifold admitting semi-symmetric metric S-connection and the form of curvature tensor $\tilde{R}$ of the manifold relative to this connection has been derived. It has been shown that if a generalized contact metric manifold admits a semi-symmetric metric S-connection whose curvature tensor is locally isometric to the unit sphere $S^n(1)$, then the con-harmonic and conformal curvature tensors with respect to the Riemannian connection are identical iff $\frac{2a^2}{c}+n=0$. Under the same condition it has been shown that con-circular curvature tensor C coincides with the curvature tensor K with respect to Riemannian connection.

Introduction

Preliminaries

If on an odd dimensional differentiable manifold $M_n (n=2m+1)$ of differentiability class C^∞, there exist a tensor Φ of the type (1,1), a vector field U, a 1-form u and a Riemannian metric G satisfying

$$\Phi^2(X)=a^2X-cu(X)U. \tag{1.1}$$

$$u(\bar{X})=0. \tag{1.2}$$

Where $\Phi(X)=\bar{X}$, c is an integer for arbitrary vector field X and a is a nonzero complex number, then M_n is said to be a generalized contact manifold and the system (Φ,U,a,c,u) is said to given a generalized contact structure to M_n.

In consequence of (1.1) and (1.2), we find

$$u(U)=\frac{a^2}{c}. \tag{1.3}$$

$$\bar{U}=0. \tag{1.4}$$

If the associated Riemannian metric G of the type (0, 2) in M_n satisfying

$$G(\bar{X},\bar{Y})=a^2G(X,Y)-cu(X)u(Y). \tag{1.5}$$

for arbitrary vector fields X,Y in M_n, then (M_n,G) is said to be generalized contact metric manifold and the structure (Φ,U,a,c,u,G) is called a generalized contact metric structure to M_n.

Putting U for X in (1.5) and then using (1.3) and (1.4), we find

$$u(X)=G(X,U). \tag{1.6}$$

Remark 1.1: A generalized contact metric structure manifold (Φ,U,a,c,u,G) Singh [13] gives an Almost Norden contact metric manifold Singh and Singh [15], an Almost para norden contact metric manifold Singh and Singh [14], an Almost para contact metric manifold Adati and Matsumoto [2], Crasmareanu [6] or Lorentzian para contact metric manifold Matsumoto [8], Bahadir [3] according as $(a^2=-1,\ c=1)$, $(a^2=-1,\ c=-1)$, $(a^2=1,\ c=1)$ or $(a^2=1,\ c=-1)$.

Definition 1.1: A C^∞-manifold, satisfying

$$D_XU=\Phi(X)\underline{\underline{def}}\ \bar{X}. \tag{1.7}$$

will be denoted by M_n^*.

Consequently in M_n^* we can shown that

$$(D_Xu)(Y)={}'\Phi(X,Y)=(D_Yu)(X). \tag{1.8}$$

where

$${}'\Phi(X,Y)\underline{\underline{def}}\ G(\bar{X},Y)=G(X,\bar{Y})={}'\Phi(Y,X). \tag{1.9}$$

Semi-symmetric metric S-connection

Definition 2.1: Let $\tilde{\nabla}$ be an affine connection is said to be metric if

$$\tilde{\nabla}_X G = 0. \tag{2.1}$$

The metric connection $\tilde{\nabla}$ satisfying

$$\left(\tilde{\nabla}_X \Phi\right)(Y) = u(Y)X - G(X,Y)U. \tag{2.2}$$

is called S-connection. A metric S-connection $\tilde{\nabla}$ is called semi-symmetric metric S-connection if

$$\tilde{\nabla}_X Y = D_X Y - u(X)\overline{Y}. \tag{2.3}$$

Where D is the Riemannian connection. Also equation (2.2) implies

$$S(X,Y) = u(Y)\overline{X} - u(X)\overline{Y}. \tag{2.4}$$

where S is the torsion tensor of connection $\tilde{\nabla}$.

Replacing Y by U in (2.2), we have

$$\left(\tilde{\nabla}_X \Phi\right)(U) = u(U)X - G(X,U)U.$$

Using (1.1), (1.3) and (1.6) in the above equation, we get

$$\left(\tilde{\nabla}_X \Phi\right)(U) = \frac{\overline{\overline{X}}}{c}. \tag{2.5}$$

In view of (1.4), we have

$$\Phi U = 0.$$

Differentiating covariantly above equation with respect to X, we get

$$\left(\tilde{\nabla}_X \Phi\right)(U) + \Phi\left(\tilde{\nabla}_X U\right) = 0.$$

Using (2.5) and $\Phi(X) = \overline{X}$ in the above equation, we get

$$\tilde{\nabla}_X U = -\frac{\overline{X}}{c}. \tag{2.6}$$

Now, from (1.6), we have

$$G(Y,U) = u(Y).$$

Taking the covariant derivative of the above equation with respect to X and using (1.9), (2.1) and (2.6), we get

$$-`\Phi(X,Y) = c\left(\tilde{\nabla}_X u\right)(Y). \tag{2.7}$$

We know that

$$\Phi Z = \overline{Z}.$$

Differentiating covariantly above equation with respect to X and using (2.3), we get

$$\left(D_X \Phi\right)(Z) = \left(\tilde{\nabla}_X \Phi\right)(Z). \tag{2.8}$$

Let $\tilde{R}$ and K be the curvature tensors with respect to the connection $\tilde{\nabla}$ and D respectively, then

$$\tilde{R}(X,Y,Z) \underline{\underline{def}} \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X,Y]} Z. \tag{2.9}$$

$$K(X,Y,Z) \underline{\underline{def}} D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z. \tag{2.10}$$

Using (2.3) and (2.10) in (2.9), we get

$$\tilde{R}(X,Y,Z) = K(X,Y,Z) - u(Y)\left(D_X \Phi\right)(Z) + \left\{\left(\tilde{\nabla}_Y u\right)(X) - \left(\tilde{\nabla}_X u\right)(Y)\right\}\overline{Z} + \left\{u\left(\tilde{\nabla}_Y X\right) - u\left(\tilde{\nabla}_X Y\right)\right\}\overline{Z} + u\left(D_X Y - D_Y X\right)\overline{Z} + u(X)\left(D_Y \Phi\right)(Z).$$

Using (1.8), (2.2), (2.3), (2.7) and (2.8) in the above equation, we get

$$\tilde{R}(X,Y,Z) = K(X,Y,Z) - u(Y)u(Z)X + u(Y)G(X,Z)U + \left\{-u\left(D_X Y\right) + u(X)u(\overline{Y}) + u\left(D_Y X\right) - u(Y)u(\overline{X})\right\}\overline{Z} + u\left(D_X Y - D_Y X\right)\overline{Z} + u(X)u(Z)Y - G(Y,Z)u(X)U.$$

Using (1.2) in the above equation, we get

$$\tilde{R}(X,Y,Z) = K(X,Y,Z) - u(Y)u(Z)X + u(Y)G(X,Z)U + u(X)u(Z)Y - u(X)G(Y,Z)U. \tag{2.11}$$

Let us consider that $\tilde{R}(X,Y,Z) = 0$, then above equation implies

$$K(X,Y,Z) = u(Y)u(Z)X - u(Y)G(X,Z)U - u(X)u(Z)Y + u(X)G(Y,Z)U. \tag{2.12}$$

Contracting above equation with respect to X, we get

$$Ric(Y,Z) = (n-2)u(Y)u(Z) + \frac{a^2}{c}G(Y,Z). \tag{2.13}$$

Contracting with respect to Z in the above equation, we get

$$rY = (n-2)u(Y)U + \frac{a^2}{c}Y. \tag{2.14}$$

Contracting Y in the above equation, we get

$$\tilde{R} = \frac{2a^2}{c}(n-1). \tag{2.15}$$

Where Ric and $\tilde{R}$ are Ricci tensor and scalar curvature respectively.

The Projective curvature tensor W, Con-harmonic curvature tensor L, Conformal curvature tensor V and Con-circular curvature tensor C in a Riemannian manifold are given by Boothby [4], Mishra [9], Pokhariyal and Mishra [10, 11, 12], Mandal and Das [7], Abood and Al-Hussaini [1], Chavan [5].

$$W(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-1)}\left[Ric(Y,Z)X - Ric(X,Z)Y\right]. \tag{2.16}$$

$$L(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-2)}\left[Ric(Y,Z)X - Ric(X,Z)Y + G(Y,Z)r(X) - G(X,Z)r(Y)\right]. \tag{2.17}$$

$$V(X,Y,Z) = K(X,Y,Z) - \frac{1}{(n-2)}\left[Ric(Y,Z)X - Ric(X,Z)Y + G(Y,Z)r(X) - G(X,Z)r(Y)\right] + \frac{\tilde{R}}{(n-1)(n-2)}\left[G(Y,Z)X - G(X,Z)Y\right]. \tag{2.18}$$

$$C(X,Y,Z) = K(X,Y,Z) - \frac{\tilde{R}}{n(n-2)}\left[G(Y,Z)X - G(X,Z)Y\right]. \tag{2.19}$$

where

$$'W(X,Y,Z,T) \underline{\underline{def}}\, G(W(X,Y,Z),T). \tag{2.20}$$

$$'L(X,Y,Z,T) \underline{\underline{def}}\, G(L(X,Y,Z),T). \tag{2.21}$$

$$'V(X,Y,Z,T) \underline{\underline{def}}\, G(V(X,Y,Z),T). \tag{2.22}$$

$$'C(X,Y,Z,T) \underline{\underline{def}}\, G(C(X,Y,Z),T). \tag{2.23}$$

Curvature tensors

Theorem 3.1: If a generalized contact metric structure manifold M_n admits a semi-symmetric metric S-connection whose curvature tensor is locally isometric to the unit sphere $S^n(1)$, then the Con-harmonic curvature tensor L and Conformal curvature tensor V with respect to the Riemannian connection are identical iff $\frac{2a^2}{c} + n = 0.$

Proof: As we know that if the curvature tensor with respect to the semi-symmetric metric S - connection is locally isometric to the unit sphere $S^n(1)$, then

$$\tilde{R}(X,Y,Z) = G(Y,Z)X - G(X,Z)Y. \tag{3.1}$$

Inconsequence of (3.1), equation (2.11) becomes

$$G(Y,Z)X - G(X,Z)Y = K(X,Y,Z) - u(Y)u(Z)X + u(Y)G(X,Z)U + u(X)u(Z)Y - u(X)G(Y,Z)U.$$

Contracting the above equation with respect to X, we get

$$Ric(Y,Z) = \left(\frac{a^2}{c} + n - 1\right)G(Y,Z) + (n-2)u(Y)u(Z). \tag{3.2}$$

Again contracting above equation with respect to Z, we get

$$rY = \left(\frac{a^2}{c} + n - 1\right)Y + (n-2)u(Y)U.$$

Now contracting above equation with respect to Y, we get

$$\tilde{R} = (n-1)\left(\frac{2a^2}{c} + n\right). \tag{3.3}$$

Where Ric and $\tilde{R}$ are Ricci tensor and scalar curvature of the manifold respectively.

We obtain the necessary part of the theorem with the help of the equation (3.3), (2.17) and (2.18). Also the converse part is obvious from (2.17) and (2.18).

Theorem 3.2: If a generalized contact metric structure manifold M_n admits a semi-symmetric metric S-connection whose curvature tensor is locally isometric to the unit sphere $S^n(1)$, then the Con-circular curvature tensor C coincides with curvature tensor K with respect to the Riemannian connection if $\frac{2a^2}{c} + n = 0.$

Proof: Using (3.3) in (2.19), we get

$$C(X,Y,Z) = K(X,Y,Z) - \left(\frac{\frac{2a^2}{c} + n}{n}\right)\left[G(Y,Z)X - G(X,Z)Y\right]. \tag{3.4}$$

which is the required proves of the theorem.

Now, let us consider that the curvature tensor of the semi-symmetric metric S-connection has the form

$$\tilde{R}(X,Y,Z) = {}'\Phi(X,Z)\bar{Y} - {}'\Phi(Y,Z)\bar{X}. \tag{3.5}$$

Using above equation in (2.11), we get

$$K(X,Y,Z)=u(Y)u(Z)X-u(X)u(Z)Y+u(X)G(Y,Z)U-u(Y)G(X,Z)U+{}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}. \tag{3.6}$$

Contracting X in the above equation and using (1.1), (1.3) and (1.6), we get

$$Ric(Y,Z)=(n-c-2)u(Y)u(Z)+\frac{a^2}{c}(c+1)G(Y,Z). \tag{3.7}$$

Contracting above equation with respect to Z, we get

$$rY=(n-c-2)u(Y)U+\frac{a^2}{c}(c+1)Y. \tag{3.8}$$

Contracting above equation with respect to Y, we get

$$\tilde{R}=\frac{a^2}{c}(n-1)(c+2). \tag{3.9}$$

Now, using (3.6) and (3.7) in (2.16), we get

$$W(X,Y,Z)={}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}+u(X)G(Y,Z)U-u(Y)G(X,Z)U+\frac{1}{n-1}\left[(c+1)u(Y)u(Z)X-(c+1)u(X)u(Z)Y\right]+\frac{a^2(c+1)}{c(n-1)}\left[G(X,Z)Y-G(Y,Z)X\right]. \tag{3.10}$$

Now operating G on both sides of above equation and using (1.6), (1.9) and (2.20), we get

$$'W(X,Y,Z,T)={}'\Phi(X,Z){}'\Phi(Y,T)-{}'\Phi(Y,Z){}'\Phi(X,T)+\left[u(X)u(T)G(Y,Z)-u(Y)u(T)G(X,Z)\right]+\frac{1}{(n-1)}\left[(c+1)u(Y)u(Z)G(X,T)-(c+1)u(X)u(Z)G(Y,T)\right]+\frac{a^2(c+1)}{c(n-1)}\left[G(X,Z)G(Y,T)-G(Y,Z)G(X,T)\right]. \tag{3.11}$$

Now, using (3.6), (3.7) and (3.8) in (2.17), we get

$$L(X,Y,Z)={}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}+\frac{1}{n-2}\left[cu(X)G(Y,Z)U-cu(Y)G(X,Z)U+cu(Y)u(Z)X-cu(X)u(Z)Y\right]+\frac{2a^2(c+1)}{c(n-2)}\left[G(X,Z)Y-G(Y,Z)X\right]. \tag{3.12}$$

Operating G on both sides of above equation and using (1.6), (1.9) and (2.21), we get

$$'L(X,Y,Z,T)={}'\Phi(X,Z){}'\Phi(Y,T)-{}'\Phi(Y,Z){}'\Phi(X,T)+\frac{1}{(n-2)}\left[cu(X)u(T)G(Y,Z)-cu(Y)u(T)G(X,Z)+cu(Y)u(Z)G(X,T)-cu(X)u(Z)G(Y,T)\right]+\frac{2a^2(c+1)}{c(n-2)}\left[G(X,Z)G(Y,T)-G(Y,Z)G(X,T)\right]. \tag{3.13}$$

Using (3.6) in (2.18), we get

$$V(X,Y,Z)={}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}+u(X)G(Y,Z)U-u(Y)G(X,Z)U+u(Y)u(Z)X-u(X)u(Z)Y-\frac{1}{(n-2)}\left[Ric(Y,Z)X-Ric(X,Z)Y-G(X,Z)r(Y)+G(Y,Z)r(X)\right]+\frac{\tilde{R}}{(n-1)(n-2)}\left[G(Y,Z)X-G(X,Z)Y\right]. \tag{3.14}$$

Using (3.7), (3.8) and (3.9) in the above equation, we get

$$V(X,Y,Z)={}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}+\frac{a^2}{(n-2)}\left[G(X,Z)Y-G(Y,Z)X\right]+\frac{1}{(n-2)}\left[cu(X)G(Y,Z)U-cu(Y)G(X,Z)U+cu(Y)u(Z)X-cu(X)u(Z)Y\right]. \tag{3.15}$$

Operating G on both sides of the above equation and using (1.6), (1.9) and (2.22), we get

$$'V(X,Y,Z,T)={}'\Phi(X,Z){}'\Phi(Y,T)-{}'\Phi(Y,Z){}'\Phi(X,T)+\frac{a^2}{(n-2)}\left[G(X,Z)G(Y,T)-G(Y,Z)G(X,T)\right]+\frac{1}{(n-2)}\left[cu(X)u(T)G(Y,Z)-cu(Y)u(T)G(X,Z)+cu(Y)u(Z)G(X,T)-cu(X)u(Z)G(Y,T)\right]. \tag{3.16}$$

Using (3.6) and (3.9) in (2.19), we get

$$C(X,Y,Z)={}'\Phi(X,Z)\bar{Y}-{}'\Phi(Y,Z)\bar{X}+\left[u(X)G(Y,Z)U-u(Y)G(X,Z)U+u(Y)u(Z)X-u(X)u(Z)Y\right]-\frac{a^2(c+2)}{cn}\left[G(Y,Z)X-G(X,Z)Y\right]. \tag{3.17}$$

Operating G on both sides of above equation and using (1.6), (1.9) and (2.23), we get

$$'C(X,Y,Z,T)={}'\Phi(X,Z){}'\Phi(Y,T)-{}'\Phi(Y,Z){}'\Phi(X,T)+\left[u(X)u(T)G(Y,Z)-u(Y)u(T)G(X,Z)+u(Y)u(Z)G(X,T)+u(X)u(Z)G(Y,T)\right]-\frac{a^2(c+2)}{cn}\left[G(Y,Z)G(X,T)-G(X,Z)G(Y,T)\right]. \tag{3.18}$$

Theorem 3.3: On a C^∞-manifold M_n, we have

$$'W(X,Y,Z,U)=\frac{a^2(n-c-2)}{c(n-1)}\left[u(X)G(Y,Z)-u(Y)G(X,Z)\right]. \tag{3.19a}$$

$$'W(U,Y,Z,T)=\left(\frac{a^2}{c}\right)\left(\frac{n-c-2}{n-1}\right)u(T)G(Y,Z)-\frac{a^2(c+1)}{(n-1)}u(Z)G(Y,T)+\left(\frac{c-n+2}{n-1}\right)u(Y)u(Z)u(T). \tag{3.19b}$$

Proof: Replacing T by U in (3.11), we get

$$'W(X,Y,Z,U)={}'\Phi(X,Z){}'\Phi(Y,U)-{}'\Phi(Y,Z){}'\Phi(X,U)+\left[u(X)u(U)G(Y,Z)-u(Y)u(U)G(X,Z)\right]+\frac{1}{(n-1)}\left[(c+1)u(Y)u(Z)G(X,U)-(c+1)u(X)u(Z)G(Y,U)\right]+\frac{a^2(c+1)}{c(n-1)}\left[G(X,Z)G(Y,U)-G(Y,Z)G(X,U)\right].$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.19a).

Replacing X by U in (3.11), we get

$$`W(U,Y,Z,T) = `\Phi(U,Z)`\Phi(Y,T) - `\Phi(Y,Z)`\Phi(U,T) + [u(U)u(T)G(Y,Z) - u(Y)u(T)G(U,Z)] + \frac{1}{(n-1)}$$

$$[(c+1)u(Y)u(Z)G(U,T) - (c+1)u(U)u(Z)G(Y,T)] + \frac{a^2(c+1)}{c(n-1)}[G(U,Z)G(Y,T) - G(Y,Z)G(U,T)]$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.19b).

Theorem 3.4: On a C^∞-manifold M_n, we have

$$`L(U,Y,Z,T) = \frac{a^2(c+2)}{c(n-2)}[u(Z)G(Y,T) - u(T)G(Y,Z)]. \quad (3.20a)$$

$$`L(X,Y,Z,U) = \frac{a^2(c+2)}{c(n-2)}[u(Y)G(X,Z) - u(X)G(Y,Z)]. \quad (3.20b)$$

Proof: Replacing X by U in (3.13), we get

$$`L(U,Y,Z,T) = `\Phi(U,Z)`\Phi(Y,T) - `\Phi(Y,Z)`\Phi(U,T)$$

$$+\frac{1}{(n-2)}[cu(U)u(T)G(Y,Z) - cu(Y)u(T)G(U,Z) + cu(Y)u(Z)G(U,T) - cu(U)u(Z)G(Y,T)] + \frac{2a^2(c+1)}{c(n-2)}[G(U,Z)G(Y,T) - G(Y,Z)G(U,T)].$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.20a).

Replacing T by U in (3.13), we get

$$`L(X,Y,Z,U) = `\Phi(X,Z)`\Phi(Y,U) - `\Phi(Y,Z)`\Phi(X,U)$$

$$+\frac{1}{(n-2)}[cu(X)u(U)G(Y,Z) - cu(Y)u(U)G(X,Z) + cu(Y)u(Z)G(X,U) - cu(X)u(Z)G(Y,U)] + \frac{2a^2(c+1)}{c(n-2)}[G(X,Z)G(Y,U) - G(Y,Z)G(X,U)].$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.20b).

Theorem 3.5: On a C^∞-manifold M_n , we have

$$`V(X,Y,Z,U) = 0. \quad (3.21a)$$

$$`V(U,Y,Z,T) = 0. \quad (3.21b)$$

Proof: Replacing T by U in (3.16)

$$`V(X,Y,Z,U) = `\Phi(X,Z)`\Phi(Y,U) - `\Phi(Y,Z)`\Phi(X,U)$$

$$+\frac{a^2}{(n-2)}[G(X,Z)G(Y,U) - G(Y,Z)G(X,U)] + \frac{1}{(n-2)}[cu(X)u(U)G(Y,Z) - cu(Y)u(U)G(X,Z) + cu(Y)u(Z)G(X,U) - cu(X)u(Z)G(Y,U)].$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.21a).

Replacing X by U in (3.16), we get

$$`V(U,Y,Z,T) = `\Phi(U,Z)`\Phi(Y,T) - `\Phi(Y,Z)`\Phi(U,T) + \frac{a^2}{(n-2)}[G(U,Z)G(Y,T) - G(Y,Z)G(U,T)] + \frac{1}{(n-2)}[cu(U)u(T)G(Y,Z) - cu(Y)u(T)G(U,Z)$$

$$+cu(Y)u(Z)G(U,T) - cu(U)u(Z)G(Y,T)].$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.21b).

Theorem 3.6: On a C^∞-manifold M_n, we have

$$`C(U,Y,Z,T) = \frac{a^2}{c}\left[\left(1 - \frac{(c+2)}{n}\right)u(T)G(Y,Z) + \left(1 + \frac{(c+2)}{n}\right)u(Z)G(Y,T)\right]. \quad (3.22a)$$

$$`C(X,Y,Z,U) = \frac{a^2}{c}\left[\left(1 - \frac{(c+2)}{n}\right)u(X)G(Y,Z) - \left(1 + \frac{(c+2)}{n}\right)u(Y)G(X,Z)\right] + 2u(X)u(Y)u(Z). \quad (3.22b)$$

Proof: Replacing X by U in (3.18), we get

$$`C(U,Y,Z,T) = `\Phi(U,Z)`\Phi(Y,T) - `\Phi(Y,Z)`\Phi(U,T) + [u(U)u(T)G(Y,Z)$$

$$-u(Y)u(T)G(U,Z) + u(Y)u(Z)G(U,T) + u(U)u(Z)G(Y,T)]$$

$$-\frac{a^2(c+2)}{cn}[G(Y,Z)G(U,T) - G(U,Z)G(Y,T)].$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.22a).

Replacing T by U in (3.18), we get

$$`C(X,Y,Z,U) = `\Phi(X,Z)`\Phi(Y,U) - `\Phi(Y,Z)`\Phi(X,U) + [u(X)u(U)G(Y,Z)$$

$$-u(Y)u(U)G(X,Z) + u(Y)u(Z)G(X,U) + u(X)u(Z)G(Y,U)]$$

$$-\frac{a^2(c+2)}{cn}[G(Y,Z)G(X,U) - G(X,Z)G(Y,U)].$$

Using (1.3), (1.4), (1.6) and (1.9) in the above equation, we get (3.22b).

Conclusion

In this work, we have revisited an Almost Norden contact metric manifold, an Almost para norden contact metric manifold, an Almost para contact metric manifold and Lorentzian para contact metric manifold. In view of the above four manifolds, we have endorsed a combined structure called Generalized contact metric structure and for the obtained structure we have been analyzed and discussed few of its properties in details.

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