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Topological indices on Clique Graph of Cyclic Subgroup Graph for Dihedral Group

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Abstract

The key objective of this present article is to look into an idea of some topological indices on clique graph of cyclic subgroup graph for dihedral group. Furthermore, we give some theorems and results in detail.

Introduction

Algebraic Graph Theory is a branch of mathematics in which graphs are constructed from the algebraic structures such as groups, rings, etc. The concept of Clique graphs were explored by Hamelink & Ronald. C. After that, Roberts and Spencer [1] have given the concept of A characterization of Clique graphs in 1971. Singh and Devi [2] have introduced the notion of Cyclic Subgroup graph for a finite group$\mathbb{Z}_n$. We extend this graph for non-abelian groups. A numerical boundary mathematically derived from the graph structure is a topological index of a graph G. There are hundreds of graph invariants with applications in many areas of Mathematics, Chemistry, Pharmacology, Social Sciences etc. Our present work is provoked by the above study and we examine some of the topological indices on clique graph of cyclic subgroup graph for dihedral group and its characteristics. Throughout this article, we discuss some theorems for prime $p, p^2$and $pq,$ where $p \ and \ q$ be distinct primes. Before entering, let us look into some necessary definitions and notations. The cyclic subgroup graph Γ_z(G) for a finite group G is a simple undirected graph in which the cyclic subgroups are vertices and two distinct subgroups are adjacent if one of them is a subset of other. For an integer $n \geq 3,$ the dihedral group D_{2n} of order $2n$ is defined by $D_{2n} =< a, f: a^n = f^2 = 1, \ faf = a^{-1} >$. The clique graph $\kappa(H)$ of an undirected simple graph H, is a graph with a vertex for each maximal clique in H. Two vertices in $\kappa(H)$ are adjacent when their corresponding maximal cliques in H share atleast a single vertex. The *Balaban Index,* Knor et al. [3] $J(H) = \frac{b}{b-a+2}\sum_{xy\in E(H)}\frac{1}{\sqrt{z(x).z(y)}}$, where a and b are the number of vertices and edges in $H, z(x)$ and$z(y)$ denotes the total of distances from x (resp. y) to every vertices of H. The *first Zagreb index index* and the *second Zagreb index,* $M_1(H) = \sum_{v\in V(G)}(d_v)^2$ and $M_2(H) = \sum_{uv\in E(G)} d_u d_v$, where d_u denotes the degree. The *First Zagreb degree eccentricity index*DE_1, $DE_1(H) = \sum_{v_i\in V(H)}(e_i + d_i)^2$. The *Second Zagreb degree eccentricity index*DE_2, $DE_2(H) = \sum_{v_i,v_j\in E(H)}(e_i + d_i)(e_j + d_j)$. Let $\sigma_n(x)$ (Kulli [4]) be the status sum of neighbour vertices. The *first status neighbourhood index,*$SN_1(H) = \sum_{xy\in E(H)}\{\sigma_n(x) + \sigma_n(y)\}$. The *second status neighbourhood index,*$SN_2(H) = \sum_{xy\in E(H)}\{\sigma_n(x).\sigma_n(y)\}$. The *Third status neighbourhood index,*$SN_3(G) = \sum_{x\in V(H)}\sigma_n(x)^2$. The *atom bond connectivity status neighbourhood index,*$ABCSN(H) = \sum_{xy\in E(H)}\sqrt{\frac{\sigma_n(x)+\sigma_n(y)-2}{\sigma_n(x).\sigma_n(y)}}$.The *atom bond connectivity status index* Kulli [5], $ABCS(G) = \sum_{xy\in E(H)}\sqrt{\frac{\sigma(x)+\sigma(y)-2}{\sigma(x).\sigma(y)}}$. The *arithmetic-geometric status index,* $AGS(H) = \sum_{xy\in E(H)}\frac{\sigma(x)+\sigma(y)}{2\sqrt{\sigma(x).\sigma(y)}}$. The *augmented status index,* $ASI(H) = \sum_{xy\in E(H)}(\frac{\sigma(x).\sigma(y)}{\sigma(x)+\sigma(y)-2})^3$. The *Forgotten topological index (F-index),*$F(H) = \sum_{x\in V(H)}(d(x))^3$. The $N_k - index, N_k(H) = \sum_{k=1}^{diam\,(H)}(\sum_{x\in V(H)} d_k(x)).k$. The *eccentric harmonic index,* $H_e(H) = \sum_{v_i,v_j\in E(H)}\frac{2}{e_i+e_j}$, The *geometric-arithmetic index,*$GA(H) = \sum_{xy\in E(H)}\frac{2\sqrt{d_x d_y}}{d_x+d_y}$. The *Randic type hadi index,* $RH(H) = \sum_{xy\in E(G)}\frac{1}{2^{d_x+d_y}}$. We give a lemma for better understanding for the readers which we have given in our paper named as Clique Graph of Cyclic Subgroup graph of dihedral Group [6].

Lemma1.1.

For a clique graph of cyclic subgroup graph on a dihedral group of order $2n, n \in N$, where $n > 2$

- $\kappa(\Gamma_z(D_{2n}))$ is isomorphic to K_{p+1} if $n = p$
- $\kappa(\Gamma_z(D_{2n}))$ is isomorphic to K_{p^2+1} if $n = p^2$
- $\kappa(\Gamma_z(D_{2n}))$ is isomorphic to K_{pq+2} if $n = pq$

Topological indices on Clique Graph of Cyclic Subgroup Graph for Dihedral Group

Theorem 2.1.

For a cyclic subgroup graph on a dihedral group of order $2n$, for $n \in \mathbb{N}$ where $n > 2$,

- $\Gamma_z(D_{2n})$, where n be prime is a tree if and only if $\Gamma_z(D_{2n})$is isomorphic to $K_{1,p+1}$
- $\Gamma_z(D_{2n})$ is unicyclic if $n = p^2$
- $\Gamma_z(D_{2n})$ is bicyclic if $n = pq$ where p,q be distinct primes.

Proof

(i) For $n = p, \Gamma_z(D_{2n})$ is a tree. The vertex set of $\Gamma_z(D_{2n})$ be

$V(\Gamma_z(D_{2n})) = \{u_1, u_2, \ldots, u_{p+2}\}$. For the cyclic subgroup graph, the cyclic sub-groups are the vertices and for two distinct subgroups are adjacent if one of them is a subset of other. Here, the vertex $u_{p+2} \in V(\Gamma_z(D_{2n}))$ consisting of a trivial subgroup (identity element) which is adjacent to all remaining u_{p+1} vertices. The another way of stating this result is a universal vertex is adjacent with every other vertex which in turn implies that $\Gamma_z(D_{2n})$ has $p+1$ pendent edges in it. Hence, for $n = p, \Gamma_z(D_{2n})$is isomorphic to $K_{1,p+1}$.

The converse is obvious.

(ii) For $n = p^2$, the vertex set of $\Gamma_z(D_{2n})$ be $V(\Gamma_z(D_{2n})) = \{u_1, u_2, \ldots, u_{p^2+3}\}$.

Here, the vertex $u_{p^2+3} \in V(\Gamma_z(D_{2n}))$ consisting of an identity element which is adjacent with every remaining u_{p^2+2} vertices. Also, u_1 and $u_2$are adjacent which makes the unicyclic graph.

(iii) For $n = pq$, the vertex set of $\Gamma_z(D_{2n})$be $V(\Gamma_z(D_{2n})) = \{u_1, u_2, \ldots, u_{pq+4}\}$. Here, the vertex $u_{pq+4} \in V(\Gamma_z(D_{2n}))$ consisting of an identity element which is adjacent with every remaining u_{pq+3} vertices. Also, the vertices $u_1, u_3 \in V(\Gamma_z(D_{2n}))$ are adjacent with u_2 whereas u_1 and u_3 are not adjacent to each other, which makes a bicyclic graph.

Theorem 2.2:

For a clique graph of cyclic subgroup graph on a dihedral group of order 2n, $n \in N$ where $n \geq 3$;

$$J\left(\kappa(\Gamma_z(D_{2n}))\right) = \begin{cases} \dfrac{(p+1)^3 - (p+1)^2}{2(p^2 - p + 2)} & if\ n = p \\ \dfrac{(p^2+1)^3 - (p^2+1)^2}{2(p^4 - p^2 + 2)} & if\ n = p^2 \\ \dfrac{(pq+2)^3 - (pq+2)^2}{2((pq)^2 - pq + 2)} & if\ n = pq \end{cases}$$

Proof:

Case i: For $n = p$

$$J(\kappa(\Gamma_z(D_{2n}))) = \frac{(p+1)^2 - (p+1)}{(p+1)^2 - (p+1) - 2(p+1) + 4}\left(\frac{(p+1)p}{2p}\right)$$

$$= \frac{(p^2+1)^2 - (p^2+1)}{p^2 - p + 2}\left(\frac{(p+1)}{2}\right)$$

$$= \frac{(p+1)^3 - (p+1)^2}{2(p^2 - p + 2)}$$

Case ii: For $n = p^2$

$$J(\kappa(\Gamma_z(D_{2n}))) = \frac{(p^2+1)^2 - (p^2+1)}{(p^2+1)^2 - 3(p^2+1) + 4}\left(\frac{(p^2+1)p^2}{2p^2}\right)$$

$$= \frac{(p^2+1)^2 - (p^2+1)}{p^4 + 2p^2 + 1 - 3p^2 - 3 + 4}\left(\frac{(p^2+1)}{2}\right)$$

$$= \frac{(p^2+1)^3 - (p^2+1)^2}{2(p^4 - p^2 + 2)}$$

Case iii: For $n = pq$

$$J(\kappa(\Gamma_z(D_{2n}))) = \frac{(pq+2)^2 - (pq+2)}{(pq+1)^2 - 3(pq+1) + 4}\left(\frac{(pq+2)(pq+1)}{2(pq+1)}\right)$$

$$= \frac{(pq+2)^2 - (pq+2)}{(pq)^2 + 4 + 2pq - 3pq - 6 + 4}\left(\frac{(pq+2)}{2}\right)$$

$$= \frac{(pq+2)^3 - (pq+2)^2}{2((pq)^2 - pq + 2)}$$

Theorem 2.3:

Let $H = \kappa(\Gamma_z(D_{2n}))$ is clique graph of a cyclic subgroup graph on $D_{2n}, n \geq 3$.

Then,

$$DE_1(H) = \begin{cases} p^3 + 3p^2 + 3p + 1 & if\ n = p \\ p^6 + 3p^4 + 3p^2 + 1 & if\ n = p^2 \\ (pq)^3 + 3(pq)^2 + 3pq + 1 & if\ n = pq \end{cases}$$

$$DE_2(H) = \begin{cases} \frac{p^4 + 3p^3 + 3p^2 + p}{2} & if\ n = p \\ \frac{p^2(p^2+1)^3}{2} & if\ n = p^2 \\ \frac{(pq+2)^3(pq+1)}{2} & if\ n = pq \end{cases}$$

Proof

Case i: For $n = p$

$$DE_1(H) = p^3 + 1 + 3p(p+1)$$
$$= p^3 + 3p^2 + 3p + 1$$

Case ii: For $n = p^2$

$$DE_1(H) = p^6 + 1 + 3p^2(p^2 + 1)$$
$$= p^6 + 3p^4 + 3p^2 + 1$$

Case iii: For $n = pq$

$$DE_1(H) = (pq)^3 + 1 + 3pq(pq + 1)$$
$$= (pq)^3 + 3(pq)^2 + 3pq + 1$$

Case i: For $n = p$

$$DE_2(H) = \frac{p(p+1)(p+1)(p+1)}{2}$$
$$= \frac{p^3 + 1 + 3p(p+1)p}{2}$$
$$= \frac{p^4 + 3p^3 + 3p^2 + p}{2}$$

Case ii: for $n = p^2$

$$DE_2(H) = \frac{(p^2+1)(p^2)(1+p^2)(1+p^2)}{2}$$
$$= \frac{(1+p^2)^3(p^2)}{2}$$

Case iii: for $n = pq$

$$DE_2(H) = \frac{(pq+2)(pq+1)(2+pq)(2+pq)}{2}$$
$$= \frac{(pq+2)^3(pq+1)}{2}$$

Theorem 2.4.

For the clique graph of cyclic subgroup on a dihedral group,

$$H_e\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = GA\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)$$
$$= \begin{cases} \dfrac{(p+1)p}{2} & if\ n = p \\ \dfrac{p^2(p^2+1)}{2} & if\ n = p^2 \\ \dfrac{(pq+2)(pq+1)}{2} & if\ n = pq \end{cases}$$

Theorem 2.5.

For the clique graph of cyclic subgroup graph on a dihedral group, the N_k-index is

$$N_k\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} (p+1)p & if\ n = p \\ p^2(p^2+1) & if\ n = p^2 \\ (pq)(pq+1) & if\ n = pq \end{cases}$$

Proof:

Case i: For $n = p$

$$N_k\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \sum_{k=1}^{diam\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)} \left(\sum_{v\in V\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)} d_k(v)\right).k$$

$$= \sum_{k=1}^{1} \left(\sum_{v\in V\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)} d(v)\right)$$

$$= p(p+1)$$

The proof is similar for $n = p^2 \& pq$

Theorem 2.6

For the clique graph of cyclic subgroup graph on a dihedral group,

$$ABCS\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} \dfrac{(p+1)\sqrt{p-1}}{\sqrt{2}} & if\ n = p \\ \dfrac{\sqrt{p^2-1}(p^2+1)}{\sqrt{2}} & if\ n = p^2 \\ \dfrac{(pq+2)\sqrt{pq}}{\sqrt{2}} & if\ n = pq \end{cases}$$

$$ABCSN\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} \dfrac{(p+1)\sqrt{(p-1)(p+1)}}{\sqrt{2}(p)} & if\ n = p \\ \dfrac{\sqrt{(p^2-1)(p^2+1)}(p^2+1)}{\sqrt{2}(p^2)} & if\ n = p^2 \\ \dfrac{(pq+2)\sqrt{(pq+2)pq}}{\sqrt{2}(pq+1)} & if\ n = pq \end{cases}$$

Proof:

Case i: For $n = p$

$$ABCS\left(\kappa\left(\Gamma_z(D_{2p})\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z(D_{2p})\right)\right)} \sqrt{\frac{\sigma(x)+\sigma(y)-2}{\sigma(x).\sigma(y)}}$$

$$= \sqrt{\frac{p+p-2}{p.p}}\left(\frac{p(p+1)}{2}\right)$$

$$= \frac{(p+1)\sqrt{p-1}}{\sqrt{2}}$$

Case ii: For $n = p^2$

$$ABCS\left(\kappa\left(\Gamma_z\left(D_{2p^2}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2p^2}\right)\right)\right)} \sqrt{\frac{\sigma(x)+\sigma(y)-2}{\sigma(x).\sigma(y)}}$$

$$= \sqrt{\frac{p^2+p^2-2}{p^2.p^2}}\left(\frac{p^2(p^2+1)}{2}\right)$$

$$= \frac{(p^2+1)\sqrt{p^2-1}}{\sqrt{2}}$$

Case iii: For $n = p$q

$$ABCS\left(\kappa\left(\Gamma_z\left(D_{2pq}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2pq}\right)\right)\right)} \sqrt{\frac{\sigma(x)+\sigma(y)-2}{\sigma(x).\sigma(y)}}$$

$$= \sqrt{\frac{(pq+1)+(pq+1)-2}{(pq+1).(pq+1)}}\left(\frac{(pq+2)(pq+1)}{2}\right)$$

$$= \frac{(pq+2)\sqrt{pq}}{\sqrt{2}}$$

Case i: For $n = p$

$$ABCSN\left(\kappa\left(\Gamma_z\left(D_{2p}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2p}\right)\right)\right)} \sqrt{\frac{\sigma_n(x)+\sigma_n(y)-2}{\sigma_n(x).\sigma_n(y)}}$$

$$= \sqrt{\frac{p^2+p^2-2}{p^2.p^2}}\left(\frac{p(p+1)}{2}\right)$$

$$= \frac{(p+1)\sqrt{(p+1)(p-1)}}{\sqrt{2}(p)}$$

Case ii: For $n = p^2$

$$ABCSN\left(\kappa\left(\Gamma_z\left(D_{2p^2}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2p^2}\right)\right)\right)} \sqrt{\frac{\sigma_n(x)+\sigma_n(y)-2}{\sigma_n(x).\sigma_n(y)}}$$

$$= \sqrt{\frac{p^4+p^4-2}{p^4.p^4}}\left(\frac{p^2(p^2+1)}{2}\right)$$

$$= \frac{(p^2+1)\sqrt{(p^2-1)(p^2+1)}}{\sqrt{2}}$$

Case iii: For $n = p$q

$$ABCSN\left(\kappa\left(\Gamma_z\left(D_{2pq}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2pq}\right)\right)\right)} \sqrt{\frac{\sigma_n(x)+\sigma_n(y)-2}{\sigma_n(x).\sigma_n(y)}}$$

$$= \sqrt{\frac{(pq+1)^2+(pq+1)^2-2}{(pq+1)^2.(pq+1)^2}}\left(\frac{(pq+2)(pq+1)}{2}\right)$$

$$= \frac{(pq+1)\sqrt{(pq+1)(pq-1)}}{\sqrt{2}(pq+1)}$$

Theorem 2.7

The forgotten index of clique graph of cyclic subgroup graph on a dihedral group,

$$F\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} p^4+p^3 & if\ n=p \\ p^8+p^6 & if\ n=\ p^2 \\ (pq+2)(pq+1)^3 & if\ n=pq \end{cases}$$

Proof:

Case i :For $n = p$

$$F\left(\kappa\left(\Gamma_z(D_{2p})\right)\right) = \ p^3+p^3\ldots.(p+1\ times)$$

$$=(p+1).p^3$$

$$=p^4+p^3$$

Case ii :For $n = p^2$

$$F\left(\kappa\left(\Gamma_z(D_{2p^2})\right)\right) = \ p^6+p^6\ldots.(p^2+1\ times)$$

$$=(p^2+1)(p^2)^3$$

$$=(p^2+1)p^6$$

Case iii :For $n = p$q

$$F\left(\kappa\left(\Gamma_z(D_{2pq})\right)\right) = \ (pq+2)+(pq+1)+\cdots(pq+2\ times)$$

$$=(pq+2).(pq+1)^3$$

Theorem 2.8.

For the clique graph of cyclic subgroup graph on a dihedral group,

$$AGS\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} \frac{(p+1)p}{2} & if\ n=p \\ \frac{p^2(p^2+1)}{2} & if\ n=\ p^2 \\ \frac{(pq+2)(pq+1)}{2} & if\ n=pq \end{cases}$$

Proof

Case i: For $n = p$

$$AGS\left(\kappa\left(\Gamma_z(D_{2p})\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z(D_{2p})\right)\right)} \frac{\sigma(x)+\sigma(y)}{2\sqrt{\sigma(x).\sigma(y)}}$$

$$= \frac{p+p}{2\sqrt{p.p}}\left(\frac{p(p+1)}{2}\right)$$

$$= \frac{(p+1)p}{2}$$

Case ii: For $n = p^2$

$$AGS\left(\kappa\left(\Gamma_z\left(D_{2p^2}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2p^2}\right)\right)\right)} \frac{\sigma(x)+\sigma(y)}{2\sqrt{\sigma(x).\sigma(y)}}$$

$$= \frac{p^2+p^2}{2\sqrt{p^2.p^2}}\left(\frac{p^2(p^2+1)}{2}\right)$$

$$= \frac{(p^2+1)p^2}{2}$$

Case iii: For $n = pq$

$$AGS\left(\kappa\left(\Gamma_z\left(D_{2pq}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2pq}\right)\right)\right)} \frac{\sigma(x)+\sigma(y)}{2\sqrt{\sigma(x).\sigma(y)}}$$

$$= \frac{(pq+1)+(pq+1)}{2\sqrt{(pq+1).(pq+1)}}\left(\frac{(pq+1)(pq+2)}{2}\right)$$

$$= \frac{(pq+1)(pq+2)}{2}$$

Theorem 2.9.

For the clique graph of cyclic subgroup graph on a dihedral group,

$$AGS\left(\kappa\left(\Gamma_z\left(D_{2n}\right)\right)\right)$$

$$= \begin{cases} \dfrac{(p+1)p^7}{16(p^3-3p^2+3p-1)} & if\ n = p \\ \dfrac{p^{14}(p^2+1)}{16(p-1)^3} & if\ n = p^2 \\ \dfrac{(pq+2)(pq+1)^7}{16(pq-1)^3} & if\ n = pq \end{cases}$$

Proof:

Case i: For $n = p$

$$ASI\left(\kappa\left(\Gamma_z\left(D_{2p}\right)\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z\left(D_{2p}\right)\right)\right)} \left(\frac{\sigma(x).\sigma(y)}{\sigma(x)+\sigma(y)-2}\right)^3$$

$$= \left(\frac{p.p}{p+p-2}\right)^3 . \frac{p(p+1)}{2}$$

$$= \frac{(p+1)p^7}{16\,(p-1)^3}$$

$$= \frac{(p+1)p^7}{16\,(p^3-3p^2+3p-1)}$$

Case ii: For $n = p^2$

$$ASI\left(\kappa\left(\Gamma_z(D_{2p^2})\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z(D_{2p^2})\right)\right)} \left(\frac{\sigma(x).\sigma(y)}{\sigma(x)+\sigma(y)-2}\right)^3$$

$$= \left(\frac{p^2.p^2}{p^2+p^2-2}\right)^3 . \frac{p^2(p^2+1)}{2}$$

$$= \frac{(p^2+1)p^{14}}{16(p^2-1)^3}$$

Case iii: For $n = pq$

$$ASI\left(\kappa\left(\Gamma_z(D_{2pq})\right)\right) = \sum_{xy\in E\left(\kappa\left(\Gamma_z(D_{2pq})\right)\right)} \left(\frac{\sigma(x).\sigma(y)}{\sigma(x)+\sigma(y)-2}\right)^3$$

$$= \left(\frac{(pq+1).(pq+1)}{(pq+1)+(pq+1)-2}\right)^3 . \frac{(pq+1)(pq+2)}{2}$$

$$= \frac{(pq+2)(pq+1)^7}{16(pq)^3}$$

Theorem 2.10.

For the clique graph of cyclic subgroup graph on a dihedral group;

$$RH\left(\kappa(\Gamma_z(D_{2n}))\right) = \begin{cases} \frac{(p+1)p}{2^{2p+1}} & if\ n = p \\ \frac{(p^2+1)p^2}{2^{2p^2+1}} & if\ n = p^2 \\ \frac{(pq+2)(pq+1)}{2^{2pq+3}} & if\ n = pq \end{cases}$$

Proof:

Case i: For $n = p$

$$RH\left(\kappa\left(\Gamma_z(D_{2p})\right)\right) = \frac{p(p+1)}{2}\left(\frac{1}{2^{p+p}}\right)$$

$$= \frac{(p+1)p}{2^{2p+1}}$$

Case ii: For $n = p^2$

$$RH\left(\kappa\left(\Gamma_z(D_{2p^2})\right)\right) = \frac{p^2(p^2+1)}{2}\left(\frac{1}{2^{p^2+p^2}}\right)$$

$$= \frac{(p^2+1)p^2}{2^{2p^2+1}}$$

Case i: For $n = pq$

$$RH\left(\kappa\left(\Gamma_z(D_{2pq})\right)\right) = \frac{(pq+2)(p+1)}{2}\left(\frac{1}{2^{pq+1+pq+1}}\right)$$

$$= \frac{(pq+2)(pq+1)}{2^{2pq+3}}$$

Theorem 2.11

For the clique graph of cyclic subgroup graph on a dihedral group,

- $SN_1\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} (p+1)p^3 & if\ n = p \\ (p^2+1)p^6 & if\ n = p^2 \\ (pq+2)(pq+1)^3 & if\ n = pq \end{cases}$
- $SN_2\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} \frac{p^5(p+1)}{2} & if\ n = p \\ \frac{p^{10}(p^2+1)}{2} & if\ n = p^2 \\ \frac{(pq+2)(pq+1)^5}{2} & if\ n = pq \end{cases}$
- $SN_3\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \begin{cases} (p+1)p^4 & if\ n = p \\ (p^2+1)p^8 & if\ n = p^2 \\ (pq+2)(pq+1)^4 & if\ n = pq \end{cases}$

Proof:

Case i : For $n = p$

$$SN_1\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) = \frac{(p^2+p^2)(p)(p+1)}{2}$$

$$= p^3(p^2+1)$$

Case ii : For $n = p^2$

$$SN_1\left(\kappa\left(\Gamma_z(D_{2p^2})\right)\right) = \frac{(p^4+p^4)(p^2)(p^2+1)}{2}$$

$$= p^6(p^2+1)$$

Case iii : For $n =$ pq

$$SN_1\left(\kappa\left(\Gamma_z(D_{2pq})\right)\right) = \frac{\{(pq+1)^2+(pq+1)^2\}(pq+2)(pq+1)}{2}$$

$$= (pq+1)^3(pq+2)$$

For (ii) & (iii), the proof is similar by (i).

Important Results

- $N_k\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) < N_k\left(\Gamma_z(D_{2n})\right)$
- $\frac{2s}{m}\sqrt{(m-1)\left(2s + M_1\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)\right) - \frac{4s^2}{m} - \frac{4s^2}{m^2}} \leq \sqrt{m\left(2s + M_1\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)\right) - \frac{4s^2}{m}}$

  where m and s represents the vertices and edges
- $M_1\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) \geq M_1\left(\Gamma_z(D_{2n})\right)$
- $M_2\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) > M_2\left(\Gamma_z(D_{2n})\right)$

- $F\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) \geq \frac{M_1\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)}{s}\left(2H\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) + M_1\left(\kappa\left(\Gamma_z(D_{2n})\right)\right)\right) - 2M_2\left(\kappa\left(\Gamma_z(D_{2n})\right)\right) - 4s$

where s represents the number of edges.

Conclusions

In this article, we have examined some topological indices on clique graph of cyclic subgroup graph for dihedral group. Moreover, we have given some theorems and results in detail.

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