SRMS Journal of Mathematical Sciences, Vol-7, 2023, pp. 24-31 **Research Article**
ISSN: 2394-725X

Pricing Strategy of an Economical Order Quantity Inventory Model for an Imperfect Quality Items Considering with Shortages and Backordering

Priyanka Singh[1], Leena Sharma[2], A. R. Nigwal[3*], Anuj Kumar[4]

[1]*Department of Mathematics, Government Degree College, Piprai, 473440, Ashoknagar, Madhya Pradesh, India.*
[2,3]*Department of Mathematics, Ujjain Engineering College, Ujjain, 456010, Madhya Pradesh, India.*
[4]*Department of Mathematics, Chandigarh University, Mohali, Punjab, India.*
**Corresponding author Email: arnwnigwal@gmail.com*

***Received:** 20 July, 2024; **Revised:** 16 August, 2024; **Accepted:** 02 September, 2024; **Published:** 28 September, 2024*

DOI: 10.29218/srmsmaths.v7i2.04

Keywords:

EOQ inventory model, screening process, imperfect items, backordering.

AMS Subject Classification:
90B05, 90B30, 90B50

Abstract

This study tackles the problem to decide a parsimonious order quantity (POQ) for highly demanded and imperfect quality items so that profit becomes maximized. Generally, it is natural to have shortage of a highly demanded items and therefore we have considered shortage and backordering policy in this study. We have studied this problem for the case, the reorder is made when the inventory level becomes zero. A screening process is applied on each lot of items to separate the items with imperfect and perfect quality.. When the supplier delivers the products to the retailer, the retailer immediately inspects all received items through the screening process. As per the ordering policy, the retailer sells these items with imperfect and perfect quality to the customers comparatively at a low and high price respectively at the one business period. For satisfying the remaining demand of the customers for items with perfect quality, the retailer buys items from nearby markets at a high buying price. In this study, we optimized one business period in the condition at which the inventory level is still positive, agglomerate size, selling price, and gross profit per unit time. The concavity property is demonstrated graphically and numerically. Moreover, sensitivity analysis has been also done to explore the positive and negative impact of model parameters on outputs.

Introduction

In real life, production practice does not lead always to perfect quality items, A possibility of a few and certain part of imperfect quality items always occurs. Moreover, a few fractions of imperfect quality items are ordinarily impending in the ordered lot size. The shortage may occur in the inventory system due to the attendance of imperfect quality and highly demanded items. Lead time may be also a common reason for occurring shortages. Harris [1] was the first scholar who suggest an EOQ inventory model. After that Rosenblatt & Lee [2] and Porteus [3], suggested their inventory models in which they considered the attendance of imperfect quality items during the manufacturing process. An interesting theoretical study has been proposed by Roy *et al.* [4], who consider an EOQ model in which a portion of imperfect quality lot size follows a uniform distribution function. An associated expected average profit function was proposed under the uniform distribution function. They also discussed the model incorporating fractional backlogging and lost sale cases for imperfect quality items.

Hsu and Hsu [5], suggested an inventory model for items with imperfect quality considering screening errors, shortage backordering, and selling return. In this study, they provided a closed-loop-form solution for economical order size, the reorder point, and an optimal backordering quantity. Dey and Giri [6], deigned an iterative algorithm to derive the optimal investment of the vendor for reducing the rate of faulty items so that the joint anticipated per annum total cost can be minimized. Jaber *et al.* [7] suggested that defective items reduce the performance of supply chains and the required treatment to change imperfect items to worsen. They developed two models for the portion of items with imperfect quality obtained by a single shipment in which the first model considers that items with imperfect quality are sent back to a reconstruction shop. While in the IInd model, they consider that items with imperfect quality are replaced by perfect ones from a near by vendor. Taleizadeh *et al.* [8] formulated an EOQ inventory model incorporating fractional backordering and analysed four various cases by taking into account the planning time when the batch of defective items comes back to the vendor's shop after the rework. Taleizadeh *et al.* [9] formulated an imperfect quality production model by incorporating backordering under an EPQ inventory model. Furthermore, they considered three various cases depending upon when the reworked items innervate into the production inventory system.

A study of the concerned neoterics literature shows that demand is met as long as monitoring is being executed, while, in practice, could be impossible. In view of the above, Maddah *et al.* [10] relaxed this imagination by demonstrating an order-overlapping "scheme that provides satisfying demand during the screening procedure from the former order". Chung *et al.* [11] formulated an EPQ model with a rework procedure at a alone-stage production system with well planned backorders. He extended the model of Cardenas-Barron [12] and received two main results for the annual total relevant cost. Asadkhani *et al.* [13] suggested four various EOQ models by incorporating the assumption with different types of imperfect quality items like salvage, reworkable, scrapable and rejected items. They proposed that learning about screening errors has an important impact on profitability. Manna *et al.* [14] designed an imperfect production inventory model that includes irregular carbon emissions under successive prepayments. Manna *et al.* [15] focuses on the production system reliability of an imperfect production inventory model for a two-layer supply chain. They assumed that the production system may be shifted from an in-control to an out-of-control state after a time which is governed by a irregular variable. An average profit function has been optimized by optimizing the production and defective rate of the production inventory system. Yu *et al.* [16] formulated a production-inventory model for a deteriorating and imperfect quality itmse. In this study the defective items are usually separated by the screening process. Only partial backordering is considered if any stage occurs shortage since not all customers are intending to wait for new inventory stocks.

Wee *et al.* [17] suggested a periodic delivery policy for their production model with vendor-buyer coordination. Chang and Ho *et al.* [18] updated the model of Wee *et al.* [19] and also applied the renewal-reward statement to derive the anticipated net profit function per unit time. Unspoiled closed-form solutions for optimal lot size, backordering quantity, and optimum anticipated profit have been obtained by using algebraic methods. Moussawi-Haidar *et al.* [20] formulated the existence of a peerless optimal lot size that maximizes the anticipated total profit. Also, they analyzed that how affected the deterioration and holding cost on the optimal batch size, backorder level, and as well as on the total profit. Paul *et al.* [21] presented a joint replenishment problem to derive an ordering stratgies for multiple items having a certain fraction of defective units. In this study, two various strategies are developed for the joint replenishment problem first one is without a price discount and the second one is with a price discount.

Rad *et al.* [22] developed an integrated/coordinated vendor and buyer supply chain model for production with imperfect allowing shortages. They assumed the market demand is selling price dependent and derived the anticipated profit per unit of time under the independent and joint optimization. Rezaei and Salimi *et al.* [23] designed an EOQ policy in which they consider screening process shifts from buyer to supplier for items with imperfect quality. Sarkar *et al.* [24] developed a theoretical model to adspection the retailer's optimal replenishment policy under warrantable latency in payment scheme with finite replenishment rate and stock-dependent demand. Skouri *et al.* [25] studies a single- congregation inventory model an EOQ with backorders to analyse the effects of supply quality under the cost performance. After screening any defects supply of batches is all accepted and then used. Taleizadeh *et al.* [26] studied an parsimonious order quantity model in the fuzzy environment for items of deteriorating nature under quantity rebate and prepayment scheme. The model was a type of mixed integer nonlinear programming, therefore solved by using Meta heuristic algorithms.

Taleizadeh *et al.* [27] suggested an EPQ model for production with imperfect quality which generates a fraction of faulty items. In this study, all faulty items are discerned by a screening procedure, and the rework of faulty items is being done by outside repair stores. Wahab *et al.* [28] presented a two-stage echelon supply chain, model to determine an EPQ for items with imperfect quality. There are three cases: the first one is both the vendor and the buyer are located in the same country, the second one is the vendor and the buyer are located in different countries, and in the third one a environmental impact is included for obtaining the EPQ under the fixed and dynamic carbon emission costs. Wee *et al.* [29] designed an EPQ model for items with imperfect quality, considering shortage and complete backorder. They also assumed that the production system produce a fraction of beggarly-quality items during the production. Imperfect quality items are picked up during the screening procdure and shelved from stock immediately.

It is needed for a inventory manager of any kind of firm to control and maintained the stocked inventories of items with perfect and imperfect quality. These type of items with perfect and imperfect quality usually occurs in many industries. For an inventory manager, the problem becomes tedious and complex if the market demand is price or time-dependent. A study of related recent literature shows one of the infirmities of models is the impractical assumption that the demand rate is constant. In this present study, we assume a demand rate is price sensitive which varies with concerned to selling price p per unit. Each received batch comprises some fraction of items with imperfect quality, with a known p. d. f. say $f(p)$. The imperfect/defective items are sold in the local nearby market at a diminished selling price per unit. In this paper, we consider an EOQ model with the requirement rate taken as a function of the selling price.

Traditionally in the economic order quantity (EOQ) models the main objective was to optimize the total profit, by minimizing the holding cost and the ordering cost ignoring the impact of price on the market demand rate. Recently, Taleizadeh *et al.* [30] suggested a model, which optimized the profit function ignoring the impact of price on market demand rate. This study develops an EOQ-type inventory model for items with imperfect quality by considering price-sensitive demand rates to determine the optimal per unit price, total profit and percentage of duration in which the inventory level is positive.

At the start of the procedure, a lot of items received from the supplier/vendor, which is enough away from the retailer's inspected rate. In the meantime, a 100% screenig process of a batch is carried out at a rate of γ, items of imperfect quality are sorted, kept in one another, and sold at a reduced rate s_c per unit before arriving the next batch. Due to permissible backordering, it is supposed that the imperfect quality items are replaced by the perfect items within the finite replenishment period N. The optimum selling price and percentage of period in which the inventory level remains positive are obtained by the optimization process to derive an optimal profit.

Notations and Assumptions

Nomenclature

The description of parameters and variables is as follows:

N: Cycle time (time unit),
T_i: Screening time of products (time/unit),
γ: I Screening rate (units/time unit),
Ω: Fractional part of imperfect quality items,
$\eta(\Omega)$: Probability density function of imperfect items,
σ : Fractional part of backordered lost sales,
l: Cost of lost sales ($/unit),
b_c: Backordered cost ($/unit),
H_e: Holding cost of necessity purchase items ($/unit/time unit),
H: Holding cost ($/unit/time unit),
o_c: Buyer'sordering cost ($/order),
p_c: Unit purchasing cost of necessity order ($/unit),
u_c: Unit cost ($/unit),
s_c: Salvaged price ($/unit),
i_c: Inspection cost ($/unit),
D_p: Demand rate (units/time unit), where $D_p = \alpha - \beta p > 0, \alpha, \beta > 0$,
p: Unit price ($/unit),
T: Percentage of length period in which inventory level is positive (time unit),
Q: Order quantity (units),
P: Total profit ($).

Assumptions

The following assumptions are supposed during the derivation of the model:

- Emergency replenishment is allowed and is equal to the items with imperfect quality for one cycle period,
- shortages, backordering are allowed,
- the demand rate is $D_p = \alpha - \beta p$ per unit/time, where α is initial demand of the item, $\alpha, \beta > 0$,
- the fractional of imperfect quality items x, and driven by the its probability density function,
- the relation $c_s < c_u < c_p$ is always maintained, for model stability,
- Inspection rate is constant and known,
- The time duration $t_1 = \frac{I_{max}}{\alpha}, I_{max}$ is maximum inventory level.
- $\alpha > D_p$, where is the inspection rate.

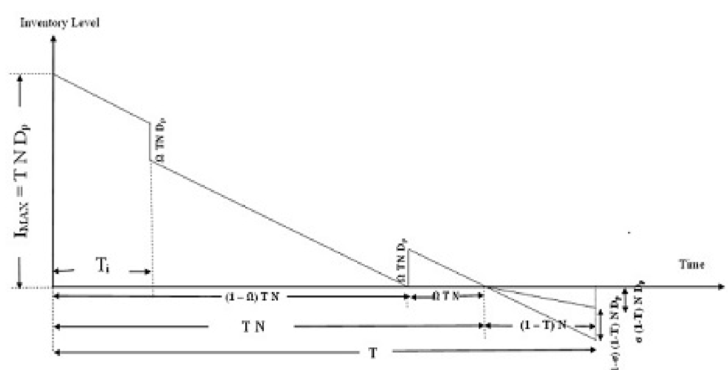

Fig. 1: Pictorial representation of Inventory level with respect to Time

Formulation of Mathematical Model

The Model

The reorder is made when the inventory level becomes zero

This paper develops a mathematical model for imperfect quality items under shortage and backordering. The reorder is made when the inventory level becomes zero. The working procedure of this model is shown in the Fig. 1. It has been supposed that a lot of TND_p units goes into the inventory system at a time $T = 0$. The process of screening is done at the rate of γ units per unit/time to detached the perfect and imperfect quality items in time duration T_i, where $T_i = \frac{TND_p}{\gamma}$. The batch contains ΩTND_p defective items percentage and $(1-\Omega)TND_p$is of items with perfect quality. After the inventory becomes zero, the imperfect items which are equal to ΩTND_p are sold immediate as a single lot at a diminished cost s_c. After the screening process, the vendor purchases ΩTND_p units from the local market at a high rate p_c to accomplish the remaining demand of the customer. The reorder is made when the inventory stock becomes zero. At the point of finishing of the cycle, the deficiency is $(1-T)ND_p.\sigma$ fraction of shortage is backordered at a charge of b_c per unit and the rest is lost sales at a charge of l per unit. The total ordered amount of items per cycle is

$$Q = TND_p + \sigma(1-T)ND_p \tag{1}$$

The logistic representation of the real life scenario is depicted in Fig. 1. The inventory holding cost percycle is calculated from Fig. 1 (summing up the areas) a s

$$C_{Hol} = H\left(\frac{(1-\Omega)^2T^2N(\alpha-\beta p)}{2} + \frac{\Omega T^2N(\alpha-\beta p)^2}{\gamma}\right) + \frac{H_e\Omega^2T^2N(\alpha-\beta p)}{2} \tag{2}$$

The shortage cost is given by

$$C_{Sho} = \frac{(1-T)^2Nb_c\sigma(\alpha-\beta p)}{2} + l(1-\sigma)(1-T)(\alpha-\beta p) \tag{3}$$

The total earned revenue per cycle is $p\{TD_p + \sigma(1-T)D_p\}$ where p is the price/unit of items. The total profit is the difference between total revenue and the total expenditure per cycle and is given as

Total Profit = sales revenue – cost of defective items – ordering cost – purchasing cost – inspection cost – emergency purchasing cost of items – holding cost of perfect and imperfect items – holding cost of emergency purchased items – cost of lost sales-backordered cost

$$\begin{aligned}P(p,T) = \Bigg[&p\{T+\sigma(1-T)\} + s_c\Omega T - \frac{O_c}{N} - u_c(T+\sigma(1-T)) - p_c\Omega T - \frac{(1-T)^2Nb_c\sigma}{2} \\ &- H\left(\frac{(1-\Omega)^2T^2N}{2} + \frac{\Omega T^2N(\alpha-\beta p)}{\gamma}\right) - l(1-\sigma)(1-T) - i_cT \\ &- \frac{H_e\Omega^2T^2N}{2}\Bigg](\alpha-\beta p)\end{aligned} \tag{4}$$

Case I

The fraction of duration in which inventory status is still positive level and when only the selling price is taken as variable.

Solution Procedure

In this case we assume the fraction of duration in which inventory status is still positive is obtained in terms of p. Then, we have to maximize $TP(p)$, where t(p) ia a function of p, satisfying the constraints $0 < \sigma \le 1, 0 < \Omega \le 1, a - bp > 0$ and $p > 0$. Now, differentiating Equation.4 with respect to T and solving equation $dP/dT = 0$, we obtained the value of T in terms of p. After substituting this value of T in Equation.4 and differentiating with respect to p we obtain

$$\begin{aligned}\frac{dP(p)}{dp} = &\frac{-2\beta HD_pH_e\gamma^2x^3(1B)^2}{N(A11)^3} - \frac{2\beta H^2D_p\gamma\Omega(1C)(1B)^2}{N(A11)^3} - \frac{D_pH_e\gamma^2(\sigma-1)\Omega^2(1B)}{N(A11)^2} \\ &- \frac{HD_p\gamma(\sigma-1)(1C)(1B)}{N(A11)^2} + \frac{\beta HD_p\gamma\Omega(1B)^2}{N(A11)^2} + \frac{\beta H_\gamma(1C)(1B)^2}{2N(A11)^2} \\ &+ \frac{2\beta i_cHD_p\gamma\Omega(1B)}{N(A11)^2} + \frac{2\beta p_cHD_p\gamma\Omega^2(1B)}{N(A11)^2} - \frac{2\beta HD_ps_c\gamma\Omega^2(1B)}{N(A11)^2} \\ &+ \frac{i_cD_p\gamma(\sigma-1)}{N(A11)} + \frac{D_ps_c\gamma(1-\sigma)\Omega}{N(A11)} + \frac{p_cD_p\gamma(\sigma-1)\Omega}{N(A11)} + \frac{\beta i_c\gamma\Omega(1D)}{N(A11)} \\ &- \frac{\beta s_c\gamma\Omega(1D)}{N(A11)} + \beta l(\sigma-1)\left[-1 - \frac{\Omega(1B)}{N(A11)}\right] + \frac{1}{2}\beta b_cN_\sigma\left[-1 - \frac{\Omega(1B)}{N(A11)}\right]^2\end{aligned}$$

$$+\left(D_p-\beta p+\beta u_c\right)\left[\sigma+\frac{\gamma\sigma(1B)}{N(A11)}-\frac{\Omega(1B)}{N(A11)}\right]-\frac{D_p b_c\gamma\sigma(1E)(1D)}{N(A11)^3}+\frac{(l+p-u_c)D_p\gamma(\sigma-1)(1D)}{N(A11)^2} \quad (5)$$

Where,

$A11 = H(1C)+\gamma(b_c\sigma+H_e\Omega^2), 1B = (i_c-p+u_c)+l(\sigma-1)+p\sigma-b_cN\sigma-u_c\sigma+p_c\Omega-s_c\Omega,$

$1C = \gamma(\Omega-1)^2+2D_p\Omega, 1D = \gamma(\sigma-1)(b_c\sigma+H_e\Omega^2)+H[\gamma(\sigma-1)(\Omega-1)^2+2x\{\alpha(\sigma-1)+\beta(i_c+u_c+l(\sigma-1)-b_cN\sigma-u_c\sigma+p_c\Omega-s_c\Omega)\}], 1E = i_c\gamma-p\gamma+HN\gamma+u_c\gamma+l\gamma(\sigma-1)+p\gamma\sigma-u_c\gamma\sigma+2D_pHN\Omega+p_c\gamma\Omega-s_c\gamma\Omega-2HN\gamma\Omega+(H+H_e)N\gamma\Omega^2$

For maximum profit

$$\frac{dP(p)}{dp}=0 \quad (6)$$

Due to the complexity of the equation, it is solved using Mathematica Software with the suitable numerical date set,

$N=0.028, \alpha=700, \beta=10, s_c=20, u_c=25, i_c=0.5, p_c=40, H_e=8, b_c=20, \sigma=97\%, \Omega=0.03, o_c=100, H=5, \gamma=175200, l=0.5$.

Then the maximum gain (profit) for the case is $P(p^*)$ is plotted agains p.

For the profit to be concave, the subsequent adequate prerequisite should be accomplished.

$$\frac{d^2P(p)}{dp^2}<0, \forall p>0 \quad (7)$$

The optimization of the profit function versus price is also acceptable graphically for this case and it is depicted in below (see Fig. 2)

Case II

The first order partial derivative of Equation.(4) with respect to p and T respectively is

$$\frac{dP(p,T)}{dp}=\beta i_cT+TD_p+\beta(1-T)l(1-\sigma)+(1-T)D_p\sigma+\frac{1}{2}\beta(1-T)^2b_cN\sigma+(p-u_c)\{-\beta T-\beta(1-T)\sigma\}+(p_c-s_c)\beta T\Omega+\frac{1}{2}\beta T^2NH_e\Omega^2-H\left[-\frac{1}{2}\beta T^2N(1-\Omega)^2-\frac{2\beta T^2D_pN\Omega}{\gamma}\right] \quad (8)$$

$$\frac{dP(p,T)}{dp}=D_p\left[-i_c+l(1-\sigma)+(1-T)b_cN\sigma+p(1-\sigma)-u_c(1-\sigma)-p_c\Omega+s_cx-TNH_e\Omega^2-H\left(TN(1-\Omega)^2+\frac{2TD_pN\Omega}{\gamma}\right)\right] \quad (9)$$

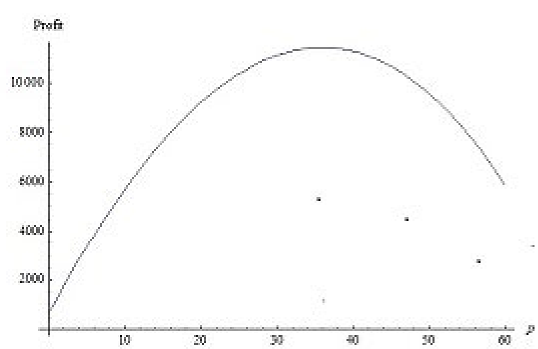

Fig. 2: Total Profit with respect to p for Case I

Solution Algorithm

Let us assume that the optimal value of $p = p^*$ and $T = T^*$ which maximizes the total profit, $P(p^*, T^*)$ may be calculated by solving the Equations.(8) and (9). consequently, the essential and adequate conditions for the total profit should be propitiated. The optimal order quantity can be obtained by inserting p^* and T^* in the Equation(1).

Taking the second order derivative of $P(p, T)$ with regard to p and T, we get

$$\frac{\partial P^2(p,T)}{\partial p^2} = -2\beta T - 2\beta(1-T)\sigma - \frac{2\beta^2 T^2 HN\Omega}{\gamma} < 0 \tag{10}$$

$$\frac{\partial P^2(p,T)}{\partial T^2} = -\frac{D_p N}{\gamma}[H\{\gamma(\Omega-1)^2 + 2D_p\Omega\} + \gamma(b_c\sigma + H_e\Omega^2)] < 0 \tag{11}$$

$$\frac{\partial P^2(p,T)}{\partial p \partial T} = \alpha + \beta i_c - \beta p + \beta(u_c - l - p)(1-\sigma) - D_p\sigma - \beta(1-T)b_c N\sigma$$

$$+(p_c - s_c)\beta\Omega + \beta TNH_e\Omega^2 H\left[\beta TN(1-\Omega)^2 + \frac{4\beta T D_p N\Omega}{\gamma}\right] \tag{12}$$

Then

$$rt - s^2 = \frac{1}{\gamma^2}[2\beta D_p N\{\gamma\sigma + T\gamma(1-\sigma) + \beta T^2 HN\Omega\}(M1) -$$
$$\{\alpha(\gamma - \gamma\sigma + 4\beta THN\Omega)$$
$$+ \beta(i_c\gamma - 2p\gamma + THN\gamma + u_c\gamma + l\gamma(\sigma-1) + 2p\gamma\sigma - b_c N\gamma\sigma$$
$$+ Tb_c N\gamma\sigma - u_c\gamma\sigma - 4\beta TH_p N\Omega + p_c\gamma\Omega - s_c\gamma\Omega - 2THN\gamma\Omega$$
$$+ THN\gamma\Omega^2 + TNH_e\gamma\Omega^2)\}^2]$$

$$\frac{\partial^2 P(p,T)}{\partial T^2}\frac{\partial^2 P(p,T)}{\partial p^2} - \left(\frac{\partial P^2(p,T)}{\partial T \partial p}\right)^2 > 0. \tag{13}$$

The concavity of the whole profit function is also acceptable graphically by the use of data given in the Example and is shown in Fig. 3.

Example

In this example we have used most of the input data from research article of Jaber*et al.*[8] and Salameh & Jaber [19], which are given below: $N = 0.028$ years, $\alpha = 700, \beta = 10, s_c = \$2$ per unit, $u_c = \$25$/unit, $i_c = \$0.5$/unit, $p_c = \$40$/unit, $H_e = \$8$/unit per year, $\sigma = 97\%, \Omega = 0.03, o_c = \$10$/order, $H = \$5$/unit per year, $\gamma = 175200$ units/year, $b_c = \$20$/ unit time, $l = \$0.5$/unit, $D_p = \alpha - \beta p$ units/year.

With the help of the above given algorithm we can obtain the optimal unit price $p^* = \$47.71$/unit, the optimal percentage of duration $T^* = 21\%$, optimal ordered quantity $Q^* = 6.09$, units, optimal total profit $TP^*(p^*, T^*) = 1278.10$ (see Fig. 3).

Table 1: Model: Optimal solutions for different cycle time, data used here are

$N = 0.028,\ \alpha = 700,\ \beta = 10,\ s_c = 20, u_c = 25,\ i_c = 0.5,\ p_c = 40,\ H_e = 8,\ \ b_c = 20,\ \sigma = 97\%,\ \Omega = 0.03,\ o_c = 100,\ H = 5, \gamma = 175200,\ l = 0.5$, *fixed* $p = 47.71,\ T = 21\%$

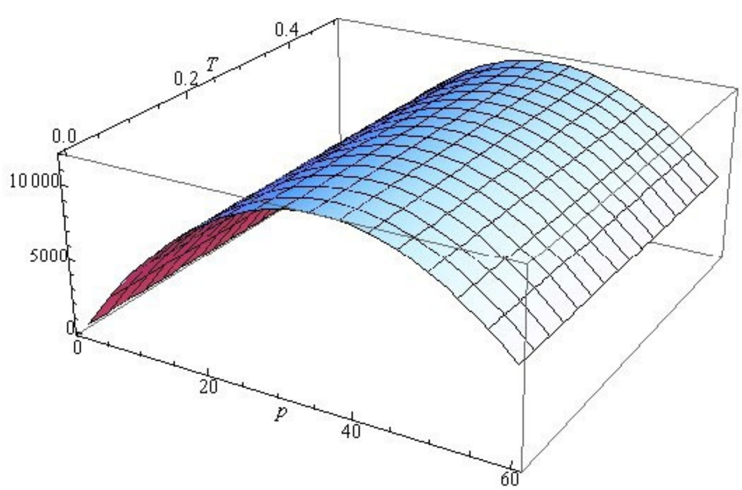

Fig. 3: Total Profit with regard to p and T for Case II

Table 1: optimal results with respect β, Q

β	Q	$TP(p,T)$
7	10.00	4392.33
8	8.70	3354.26
9	7.40	2316.18
10	6.09	1278.10
11	4.79	240.02

Table 2: Impact of Cycle Time on optimal parameter

N	Q	T	$TP(p,T)$
—20%	—78%	4%	314.00
—10%	—89%	13%	854.03
+50%	+150%	41%	2453.80
+60%	+161%	44%	2610.10
+70%	+172%	46 %	2746.70
+80%	+180%	47%	2828.58

Sensitivity Analysis

Recognizable analysis is executed based on the key parameters to established the impact on the order quantity, the fraction of duration time of the whole period in which inventory level is still positive, and vendor's whole profit. It can be perceived that from Table 1, β the value of price sensitive parameter β is negative correlated to the vendor's total worth due to demand rate per unit time. Also, ordering quantity decreases gradually. Further from Table 2, we observe that if the cycle time N varies−20%,−10%,+50%,+60%,+70%, +80%, then price will remain unchanged. the fraction of duration time of the whole period in which inventory level is positive, T increases, and the vendor's whole worth increases.

Conclusion

Supply and managing inventory of highly-demanded and essential items is very tedious and cumbersome. The problem becomes very challenging when the ordered batch contains some imperfect quality items. In the present study, we have designed parsimonious order quantity inventory model for the imperfect quality of highly demanded items. It is natural to arise a shortage of these types of items, so we have considered a backordering policy under price-dependent demand. Furthermore, in this study, we supposed that the reorder of items with perfect-quality is made when the inventory level becomes zero. We have analyzed this theoretical model's performance with respect to some key parameters and concluded that the highly demanded items are more price sensitive. Hence the vendor needs to be more careful about the pricing policy. The study suggested that businesses of highly demanded items may be more beneficial if they should be run long time duration. This study can be extended by incorporating disruption in the inventory system. It can be also elaborated by incorporating the existence of imperfect quality items in a fuzzy environment. One can be also extended this model by considering partial backlogging.

References

1. Harris F.W.,(1913).How many parts to make at once. Factory,The magazine of management, **10**(2), 135-136,152.
2. Rosenblatt M.J., Lee H.L., (1986). Economic production cycles with imperfect production processes. IIE Transactions, **18**(1), 48-55
3. Porteus E.L. (1986), Optimal lot sizing, process quality improvement and setup cost reduction. Operations Research, **34**(1), 137-144.
4. Roy M.D., Sana S.S., Chaudhuri K., (2011). An economic order quantity model of imperfect quality items with partial backlogging. International Journal of Systems Science, **42**(8), 1409-1419.
5. Hsu J.T., Hsu L.F., (2013a). Two EPQ models with imperfect production processes, inspection errors, planned backorders, and sales returns. Computers and Industrial Engineering, **64**(1), 389-402.
6. Dey O., Giri B. C.,(2014). Optimal vendor investment for reducing defect rate in a vendor buyer integrated system with imperfect production process. International Journal of Production Economics, **155**, 222-228.
7. Jaber M.Y., Zanoni S., Zavanella L.E., (2014).Economic order quantity models for imperfect items with buy and repair options. International Journal of Production Economics, **155**, 126-131.
8. Taleizadeh A.A., Khanbaglo M.P.S., Cárdenas-Barrón L.E., (2016). An EOQ inventory model with partial backordering and reparation o f imperfect products. International Journal of Production Economics, **182**, 418-434.
9. Taleizadeh A.A., Khanbaglo M. P. S., Cárdenas-Barrón L.E. (2016), An EOQ inventory model with partial backordering and reparation of imperfect products. International Journal Production Economics, **182**, 418- 434.

10. Maddah B., Salameh M.K., Moussawi-Haidar L. (2010), Order overlapping: A practical approach for preventing shortages during screening. Computers and Industrial Engineering, **58**(4), 691-695.
11. Chung K. J. (2011),The economic production quantity with rework process in supply chain management. Computers and Mathematics with Applications, **62**(6), 2547-2550.
12. Cárdenas-Barrón L. E. (2009), Economic production quantity with rework process at a sin-gle-stage manufacturing system with planned backorders. Computers and Industrial Engineering, **57**(3),1105-1113.
13. Asadkhani J., Mokhtari H., Tahmasebpoor S., (2021). Optimal lot-sizing under learning effectin inspection errors with different types of imperfect quality items, **22**(3), 2631-2665.
14. Manna A.K., Das B., Tiwari S., (2020). Impact of carbon emission on imperfect production inventory system with advance payment base free transportation. RAIRO - Operations Research, **54**(4),1103-1117.
15. Manna A.K., Dey J.K., Mondal S.K. (2017), Two layers supply chain in an imperfect production inventory model with two storage facilities under reliability consideration, Journal of Industrial and Production Engineering, **35**(2), 57-73.
16. Yu J. C. P., Wee H.M., Chen J.M. (2005), Optimal ordering policy for a deteriorating item with imperfect quality and partial backordering. Journal of the Chinese Institute of Industrial Engineers,**22**(6),509-520.
17. Wee H.M., Yu J.C.P., Wang K.J., (2006). An integrated production-inventory model for deteriorating items with imperfect quality and shortage backordering considerations. Lecture Notes in Computer Science book series, **3982**, 885-897.
18. Chang H.C., Ho C.-H. (2010), Exact closed-form solutions for optimal inventory model for items with imperfect quality and shortage backordering. Omega, **38**(3-4), 233-237.
19. Wee H., Yu J., Chen M.C., (2007). Optimal inventory model for items with imperfect quality and shortage backordering. Omega, **35**(1), 7-11.
20. Moussawi-Haidar L., Salameh M., Nasr W. (2014), Effect of deterioration on the instantaneous replenishment model with imperfect quality items. Applied Mathematical Modelling, **38**(24), 5956-5966.
21. Paul S., Wahab M.I.M. and Ongkunaruk P.(2014), Joint replenishment with imperfect items and price discount. Computers & Industrial Engineering **53**: 179 - 185.
22. Rad M.A.,Khoshalhan F. and Glock C.H. (2014),Optimizing inventory and sales decisions in a two-stage supply chain with imperfect production and backorders. Computers & Industrial Engineering **74**, 219 – 227.
23. Rezaei J. and Salimi N.(2012), Economic order quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. International Journal of Production Economics **137**(1), 11 - 18.
24. Sarkar B.(2012), An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. Applied Mathematics and Computation **218** (17) 8295 -8308.
25. Skouri K., Konstantaras I., Lagodimos A.G. and Papachristos S.(2014), An EOQ model with backorders and rejection of defective supply batches. International Journal of Production Economics, **155**, 148 - 154.
26. Taleizadeh A.A., Wee H.M. and Jolai F. (2013), Revisiting a fuzzy rough economic or-der quantity model for deteriorating items considering quantity discount and prepay- ment. Mathematical and Computer Modelling, **57** (5-6) 1466 - 1479.
27. Taleizadeh A.A., Khanbaglo M.P.S., Cárdenas-Barrón L.E., (2017). Outsourcing rework of im- perfect items in the economic production quantity (EPQ) inventory model with backordered demand. IEEE Transactions on Systems, Man. And Cybernetics: Systems, **49**(12), 2688-2699.
28. Wahab M.I.M., Mamun S.M.H. and Ongkunaruk P.(2011),EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact. International Journal of Production Economics **134** (1), 151 - 158.
29. Wee H.M., Yu J.C.P., Wang K.J., (2006). An integrated production-inventory model for deteriorating items with imperfect quality and shortage backordering considerations. Lecture Notes in Computer Science book series, **3982**, 885-897.
30. Taleizadeh A.A., Khanbaglo M.P.S. and Cárdenas-Barrón L.E. (2020), Replenishment of imperfect items in an eoq inventory model with partial backordering. RAIRO Operations Research, **54**, 413-434.